LECTURE 2

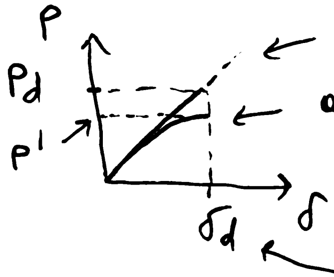
Yaoundé, 18/11/2014

Page 1

RECAP: in most cases, we have to deal with statically indeterminate structures, wich cannot be solved (i.e. reactions and internal forces found) only using equilibrium conditions applied to the structure considered as a system of rigid bodies.

A common approach for system of beams (i.e. beams but also columns, frames, etc.) ~~made~~ is to rely on a linear elastic analysis to derive the additional compatibility conditions needed to solve indeterminate structures and to found, this way, reactions and internal forces, and then deformations (curvature, rotations, displacements).

While the calculated deformations can be used (with proper adjustments) for considerations about the structure in service, in use (since it has to remain in elastic conditions!), in ultimate conditions forces ~~are~~ used for the design and the assessment ~~are~~ should be different from those derived from a linear elastic analysis, since the structural behaviour is no more linear. However, in most practical cases (with a due caution) reactions and internal forces used for design are those calculated on the basis of an elastic behaviour since they are, generally (not always!) greater than the actual forces that correspond to the ultimate conditions under verification



linear behaviour

actual behaviour (non-linear)

displacement corresponding to the design force P_d, which is greater than the actual force P' when the structure reaches the ultimate displacement δ_d

Pay attention: this is not always true, and in some cases (for example, when dealing with instability), non-linear effects must be taken into account

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Typical procedure when using a linear elastic approach to a system of beams (force method) that is statically indeterminate:

1 – Apply the equilibrium conditions, and find reactions and internal forces with the unknown values;
2 – Find rotation and displacements, then apply compatibility conditions (congruence) in order to find the only compatible configuration among the ∞^i equilibrated configurations;
3 – From M and N find χ and ε_a ($\chi(x) = M(x)/K_f$; $\varepsilon_a(x) = N(x)/K_a$)
4 – find the deformed shape by integration or other methods ($\varphi(x) = -dv'(x)$; $\chi(x) = -v''(x)$, $\varepsilon_a(x) = w'(x)$)

LIMIT STATE DESIGN

This approach to the structural design is based on a probabilistic point of view combined with some deterministic assumptions.

In general, two main classes of limit states (i.e. specified conditions that can be reached by the structure) are defined: Ultimate Limit States (ULS) and Serviceability Limit States (SLS).

ULS are related to critical conditions where safety ~~and~~ of people and structural integrity are at risk, while SLS are related to conditions that involve a satisfactory use of the structure.

ULS and SLS comprise various different groups of conditions

MAIN ULS
- EQU → related to possible loss of stability and equilibrium of the structure
- STR → include the most common strength verifications
- GEO → related to geotechnics and foundations
- FAT → related to fatigue (i.e. cyclic loading)

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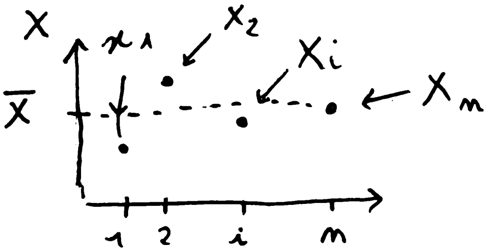
TYPICAL SLS
- deflection (excessive)
- cracking (local damage, spalling, etc...)
- durability
- excessive vibrations
- etc.

The concept of characteristic and design value

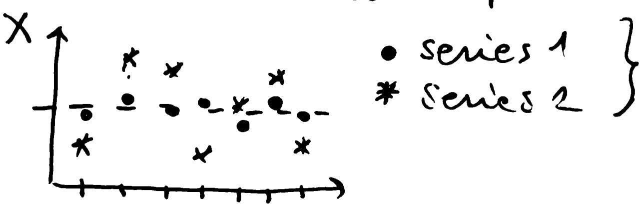
When dealing with quantities that can be measured, like the strength of a material or the intensity of a force (e.g. the wind pressure, etc.), you have the problem of choosing a representative value for that quantity.
In other words, if you repeat the measurement of the same quantity for several times, values can be variously scattered, due to various reasons (local variations, errors, etc...)
Let assume X as a generic aleatory variable (for example, the compressive strength of a certain concrete).
If you repeat n time the measure, you will get $\{X_1, X_2, \dots, X_n\}$ values.



The first representative value that could be taken into account is the average (mean) value $\overline{X}$

average (mean) value $\overline{X} = \dfrac{\sum_i X_i}{n}$, but it does not tell anything about the distribution of data.

For example:

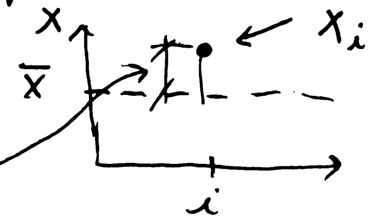


they have the same average value, but dispersion is greater for series 2

The problem is that, for your analyses and your design, you need representative values of properties (like strength, etc.) and actions (loads, etc.), so it is important to understand what is the reliability of your representative values.

Another quantity that can help in this aspect is the standard deviation (σ_{st})

$$\sigma_{st} = \sqrt{\frac{\sum (X_i - \overline{X}_m)^2}{n-1}}$$



this is $X_i - \overline{X}_m$

It is, more or less, the average of the square errors

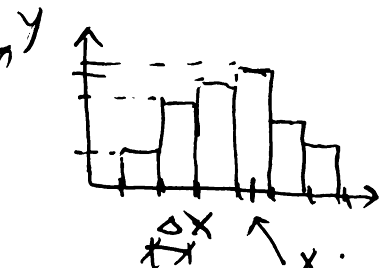
why $(X_i - \overline{X})^2$? Because, by definition, the sum $\sum (x_i - \overline{x}) = 0$; indeed, negative distances compensate positive distances.

why $(n-1)$ instead of n? There are statistical reasons, but, in practice, it avoids the calculation of σ_{st} for a series made of a single value ($\rightarrow n-1=0$!)

if the number of measures n is sufficiently high, we can subdivide the possible values of our quantity in ranges, and count how many measures fall inside each range.

Relative frequency $y = \dfrac{n_i}{n}$ $n_i \rightarrow$ number of measures within a certain range; $n \rightarrow$ overall number of measures

$\rightarrow$ Histogram of the relative frequencies



relative frequencies

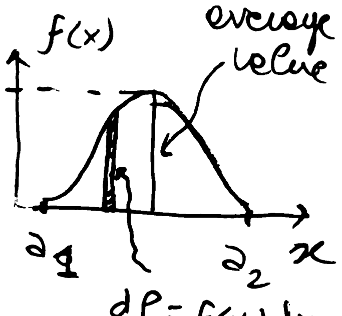
$X \rightsquigarrow$ our measured variable

X_i

Each range can be denoted by a central value X_i and by its amplitude ΔX

if $\Delta X \rightarrow 0$, we get the density of probability $f(X)$ of a certain value X, which tells us what is the probability that our quantity be equal to the value X.

This probability, dP, is given by $f(x)\,dX$



NOTE Let's use lowercase x now, assuming a continuous variation of our variable

$a_1, a_2 \rightarrow$ minimum and maximum value of our variable, respectively

$$dP = f(x)\,dx$$

The probability P_{x_0,x_1} of getting a value for a measure of our quantity that is comprised between x_0 and x_1 is given by the related integral of $f(x)$

$$P_{x_0,x_1} = \int_{x_0}^{x_1} f(x)\,dx$$

The probability P_1 of getting a measured value lower than a certain value x_K is given by:

$$P_1(x \le x_K) = \int_{a_1}^{x_K} f(x)\,dx$$

The probability P_2 of getting a measured value greater than the value x_K is given by:

$$P_2(x > x_K) = \int_{x_K}^{a_2} f(x)\,dx$$

Of course, $P_1 + P_2 = 1 \rightarrow$

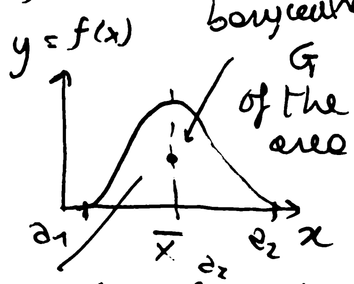
$$P_1 + P_2 = \int_{a_1}^{a_2} f(x)\,dx = 1$$

By definition, the average value $\bar{x}$ is equidistant from the other portions of the area described by $f(x)$

Therefore

$$x_m \cdot \underbrace{\int_{a_1}^{a_2} f(x)\,dx}_{\text{area } A} = \underbrace{\int_{a_1}^{a_2} x \cdot f(x)\,dx}_{\text{static moment about } y\ (S_y)}$$

$$\Rightarrow x_m = \frac{S_y}{A}$$



area $A = \int_{a_1}^{a_2} f(x)\,dx$

Standard deviation of x, now indicated by σ_x

$$\sigma_x = \sqrt{\int_{a_1}^{a_2} (x - \bar{x}_{m})^2 f(x)\,dx} \rightarrow \text{divide by } 1 = \int_{a_1}^{a_2} f(x)\,dx \rightarrow \sigma_x = \sqrt{\frac{\int_{a_1}^{a_2} (x-\bar{x})^2 f(x)\,dx}{\int_{a_1}^{a_2} f(x)\,dx}}$$

Numerator: it is the second moment of area of the area described by $f(x)$, about the barycentric axis that crosses $\bar{x}$ $\longrightarrow I_{y,G}$

Denominator: it is the area described by $f(x) \longrightarrow A$

$$\Rightarrow \text{standard deviation} \quad \sigma_x = \sqrt{\frac{I_{y,G}}{A}} = i_{y,G} \leftarrow \text{radius of gyration!}$$

Low $\sigma_x \rightarrow$ low dispersion $\rightarrow$ low $i_{y,G} \rightarrow$ area concentrated around the average value $\bar{x}$

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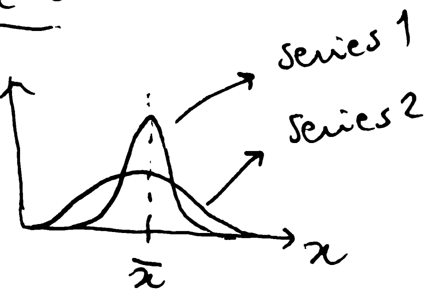
Example:



They have the same ~~equal~~ average value $\bar{x}$, but series 1 has a lower σ_x, that means a lower dispersion of data and, therefore, a greater "reliability" of the average value, from the point of view of the "representativeness".

Approximation for common engineering applications:

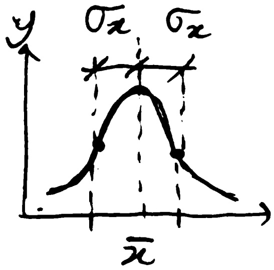
GAUSS CURVE

$$y(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \cdot e^{-\left(\frac{x-\bar{x}}{\sigma_x \sqrt{2}}\right)^2}$$

$\bar{x} \rightarrow$ average value

$\sigma_x \rightarrow$ standard deviation

It is symmetric about $\bar{x}$, which is the point having maximum probability, and has two inflexion points (where the curvature is zero and changes its sign before and after) at a distance σ_x from $\bar{x}$



Using more than 30 specimens / measures, it is possible to get a good approximation of the Gauss curve.

Percentiles: the i-percentile value of our properties is a certain ~~f~~ value that has a probability equal to i % of being not overcome.

For example, the 10-percentile value corresponds to a probability of 10% of getting a measured value that is inferior to it (and, therefore, a 90% of probability of measuring a greater value).

The 5-percentile and the 95-percentile are commonly used for strengths (the first) and for actions (the latter), since we want to use, in our calculations, values for strengths that ~~are lower~~ correspond to a low probability of a real value that is inferior, while for external forces we want to use values that correspond to a great probability of being greater than the actual values.

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The i-percentile values can be derived from the average value $\overline{x}$ and from the standard deviation σ_x

$$i\text{-percentile value} = \overline{x} - K \cdot \sigma_x$$

K depends on the chosen percentage: 5% → K = 1.64

95% → K = −1.64

the curve is symmetric!

thus

$$\text{5-percentile} = \overline{x}_m - 1.64\,\sigma_x$$
$$\text{95-percentile} = \overline{x} + 1.64\,\sigma_x$$

they are frequently called "characteristic" values

other common values:

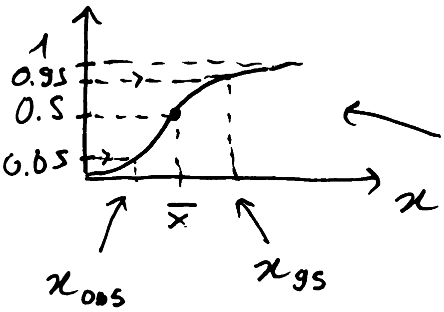
1.5-percentile → K = 2.16

(98.5-percentile → K = −2.16)

0.5-percentile → K = 2.58

(99.5-percentile → K = −2.58)

$$F(x) = \int f(x)\,dx$$



F(x) is the cumulated probability

PARTIAL FACTORS OF SAFETY

To get design values, we start from characteristic values, but we have to apply other coefficients that reduce or increase the given probability. Those coefficients are called partial factors of safety, and are denoted with γ.

Design property for materials:

$$X_d = \frac{X_K}{\gamma_M}$$

X_K → characteristic value (5-percentile)

X_d ← design value

γ_M → material partial factor

(γ reduces the strength!)

	Persistent / transient situations		accidental situations	
	CONCRETE	STEEL	CONCRETE	STEEL
ULS in flexure, shear, bond →	1.50	1.15	1.20	1.00
SLS →	1.00	1.00	NOT APPLICABLE	

Design value for actions

(γ increases the force!)

$$F_d = \gamma_F \cdot F_K$$

F_K → characteristic value (or, anyway, a representative value)

F_d → design value

γ_F → global partial factors

NOTE 1 : for actions, differently from strengths, it is not always possible to refer to existing or reliable statistical analyses of the values to be considered.
F_K, characteristic value for a generic action, does not always consists in a real 95-percentile value, but could be a nominal (i.e. deterministic and not probabilistic) value → this is one of the reasons why the state limit design is usually described as a semi-probabilistic method (not entirely based on a probabilistic approach)

NOTE 2 : in practice, a single partial factor γ_F is used for actions, but, in theory, it includes the meaning of two different safety factors.
Indeed, we have two main sources of uncertainty when dealing with forces and with the effects caused by those forces

$$\sum \text{forces} \rightarrow \text{Effects}(\text{forces})$$

The first, is related to the values of a certain force to be taken into account (as for strengths), but the second is related to the simplified models that we have to adopt in order to calculate the effects of forces and to calculate the proper capacities/resistances.
In theory, we should use something like:

$$\gamma_{model} \cdot \text{Effects}(\gamma_f \cdot \text{forces})$$

where γ_f accounts for an increment of forces, and γ_{model} accounts for the uncertainty related to the adopted models
In practice, we have just one coefficient $\gamma_F \rightarrow \gamma_F \cdot \text{forces}$

Page 9 Partial factors for permanent actions $G_K \rightarrow \gamma_G$
" " for variable actions $Q_K \rightarrow \gamma_Q$

If the effects of a certain action are favourable, the action should be reduced in order to minimize its positive influence.
If the effects of a certain action are unfavourable, that action should be increased in order to maximize its negative influence.
The ratio is that our design situation, to be on the safe side, should be reasonably worse than the possible actual situation.

	γ_G (permanent)		γ_Q (variable)	
	Favourable	Unfavourable	Favourable	Unfavourable
ULS – EQU	0.9	1.1	0	1.50
ULS – STR	1.0	1.35	0	1.50

COMBINATIONS OF ACTIONS

First, we have to introduce the ψ factors, which multiply characteristic values of variable loads Q_K to give specific values known as:

$\psi_0 \cdot Q_K \rightarrow$ combination value
$\psi_1 \cdot Q_K \rightarrow$ frequent value
$\psi_2 \cdot Q_K \rightarrow$ quasi-permanent value

NOTE: ψ factors apply only to variable actions!

Combination, frequent and quasi-permanent values are related to certain probabilities of occurrence during the working life of the structure.

EXAMPLE VALUES		ψ_0	ψ_1	ψ_2
imposed loading on ~~[illegible]~~ buildings	domestic/office	0.7	0.5	0.3
	congregation/shopping	0.7	0.7	0.6
	storage areas	1.0	0.9	0.8
	wind	0.5	0.2	0

Then, we can introduce the symbolic (not arithmetic!) combinations that have to be used to calculate the effects of actions to the structure. Prestressing actions P_K are omitted.

→ Ultimate Limit States (ULS)

FUNDAMENTAL COMBINATION $\sum_{j\geq 1} \gamma_{G,j} \cdot G_{k,j} + \gamma_{Q,1} \cdot Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i}$

It considers the permanent actions multiplied (increased or not) by γ_G (the proper ones related to each permanent load), then it considers one of the variable loads $Q_{k1}, Q_{k2}, \dots$ (if more than one), in turn (that is, the first, then the second, then the third, etc.), as the leading variable load, multiplied by its γ_Q, and takes the remaining variable actions with their combination value (then with a lower value) multiplied by the related γ_Q (accompanying actions). This accounts for a reduced probability of having all the variable loads together with their maximum values at the same time.

~~A~~ ACCIDENTAL COMBINATION

A_d → accidental action (explosion, impact of a vehicle, etc.)

$$\sum_{j\geq 1} G_{k,j} + A_d + \gamma_{Q,1} \cdot Q_{k,1} \cdot (\psi_{1,1} \text{ or } \psi_{2,1}) + \sum_{i>1} \psi_{2,i} \cdot Q_{k,i}$$

It considers the characteristic values of permanent loads (without γ_G), ~~and~~ frequent or quasi-permanent values for the leading variable load, while the accompanying variable loads are considered with their quasi-permanent values.

For earthquakes (A_{Ed}) → $A_{Ed} + \sum_{j\geq 1} G_{k,i} + \sum_{i\geq 1} \psi_{2,i} Q_{k,i}$

→ Serviceability Limit States

There are three combinations: the characteristic combination is used for irreversible SLS (where deformations, ~~...~~ etc. remain after the removal of the load), the frequent combination and the quasi-permanent one, used for reversible SLS.

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LECTURE 2

Yaounde, 18/11/2014

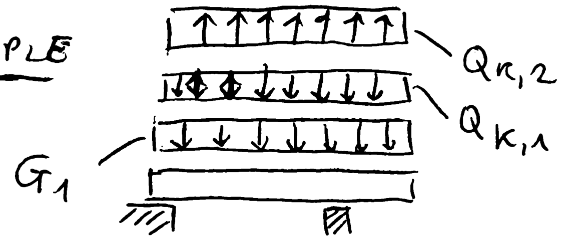
Page 11

characteristic comb. $\sum_{j\geq 1} G_{k,j} + Q_{k,1} + \sum_{i>1} \psi_{0,i}\, Q_{k,i}$

frequent comb. $\sum_{j\geq 1} G_{k,j} + \psi_{1,1}\, Q_{k,1} + \sum_{i>1} \psi_{2,i}\cdot Q_{k,i}$

quasi-permanent comb. $\sum_{j\geq 1} G_{k,j} + \sum_{i\geq 1} \psi_{2,i}\cdot Q_{k,i}$

EXAMPLE

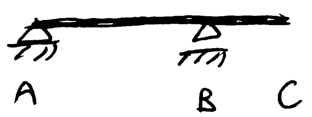


$G_1 \rightarrow$ self weight + permanent weights

$Q_{k,1} \rightarrow$ imposed loading (acting downward)

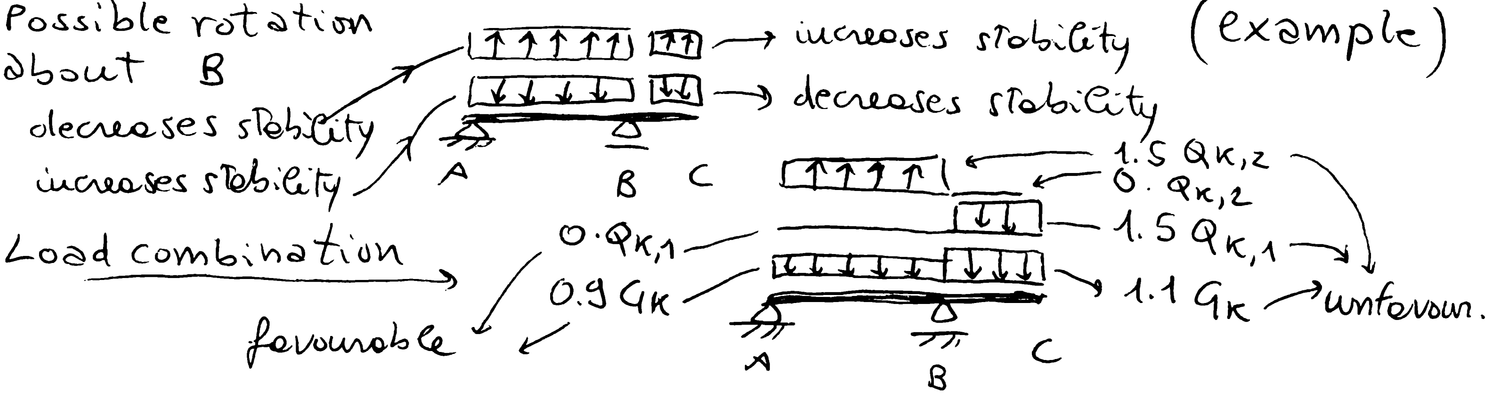
$Q_{k,2} \rightarrow$ wind action (acting upward)

Design sketch



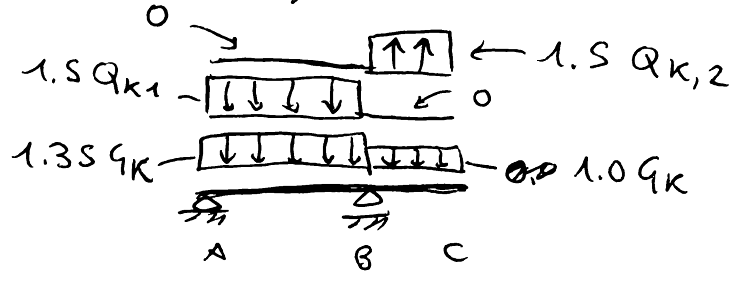
Since there is an action (wind) acting upward, and since there is a protruding part, the possible loss of equilibrium should be taken into account

- EQU ULS $\rightarrow$ check the stability of the structure considered as a system of rigid bodies, and use the proper partial factors γ_G and γ_Q.



- STR ULS $\rightarrow$ check the capacity of the structure, with reference to strength, etc.

example : check the bending strength in the span AB, that is maximise the bending moment in AB



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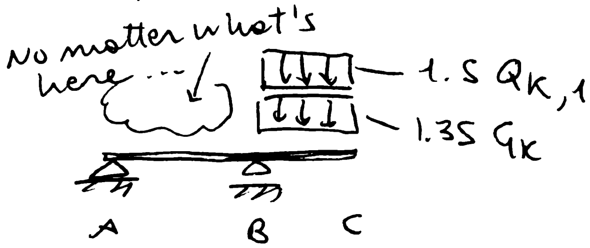
example : maximise the moment (negative) at the support B

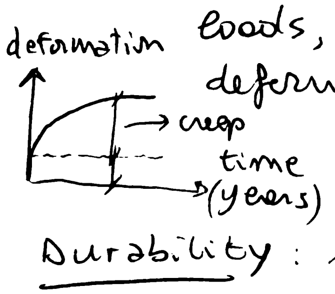


BASICS OF REINFORCED CONCRETE BEHAVIOUR (Recap)

Concrete has a good compressive strength, but its tensile strength is about 1/10 of the compressive one.
Therefore, steel reinforcement is added in order to withstand tensile forces. The collaboration between steel and concrete relies on various aspects: first, a development of bond mechanisms allows for a stress transfer between concrete and steel; then, their coefficients of thermal expansions are pretty similar ($\alpha_{T,\text{concrete}} \approx 10 \cdot 10^{-6}$ 1/°C, while $\alpha_{T,\text{steel}} \approx 7 \div 12 \cdot 10^{-6}$ 1/°C), therefore seldom problems related to differential expansions arise; moreover, the relative high pH of concrete has a good effect on the steel, that is therefore protected from corrosion (basic environment).
Perfect bond means that we can consider an equal strain for the steel and the surrounding concrete.

Shrinkage : it is a reduction in volume of the concrete during hardening, due to water absorption by cement and aggregates, and by water evaporation through the external surfaces. It can cause cracking of concrete, and has a relation with the tightening of bond with steel.

Creep: the long-term deformation of concrete, under almost constant loads, are relevant (up to 3-4 times the instantaneous deformation).



the long term deformation is proportional to $\left\{ \begin{array}{l} \text{load} \\ 1/f_c \end{array} \right.$
it will not recover after load removal

Durability : it is related to $\left\{ \begin{array}{l} \text{exposure (pollution, sea water, etc.)} \\ \text{type of cement} \\ \text{concrete quality} \\ \text{cover} \\ \text{crack width} \end{array} \right.$

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