LECTURE 3

Yaoundé, 20/11/2014

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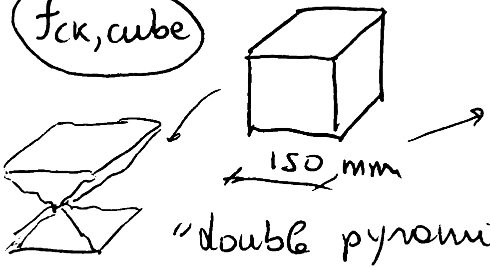
DESIGN PROPERTIES AND MATERIALS LAW FOR STEEL AND CONCRETE

CONCRETE basic property: cylindric compressive strength f_{ck} (characteristic value, 5-percentile)

cubic compressive strength → tested on cubic specimens, typically having an edge of 150 - 200 mm

$f_{ck,cube}$

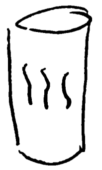


greater than this in case of certain types of concrete (i.e. with big aggregates)

"double pyramid" failure due to the friction of the plates of the test machine, which exerts a confinement ~~which~~ that improves strength.

cylindric compressive strength → tested on cylindric or prismatic specimens, typically having a height about twice the base

failure:

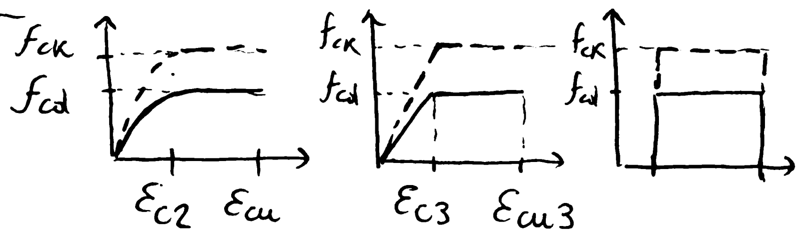


it indicates that compression was more or less unidirectional

$$\boxed{f_{ck} \cong 0.83\, f_{ck,cube}}$$

CONCRETE STRESS - STRAIN LAWS

three alternative laws



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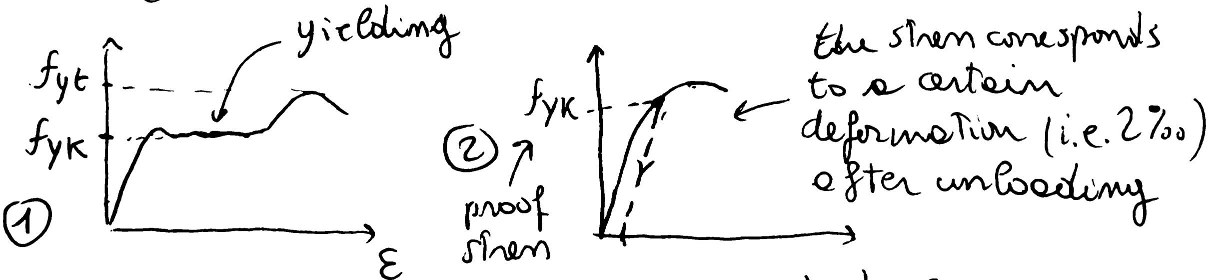
STEEL the elastic modulus of steel is generally comprised between 200 and 210 GPa (kN/mm^2)

Eurocode 2 (EC2) provides $E_s = 200$ GPa

Stress-strain behaviour in tension



hot rolled steel (rebars) — cold worked steel

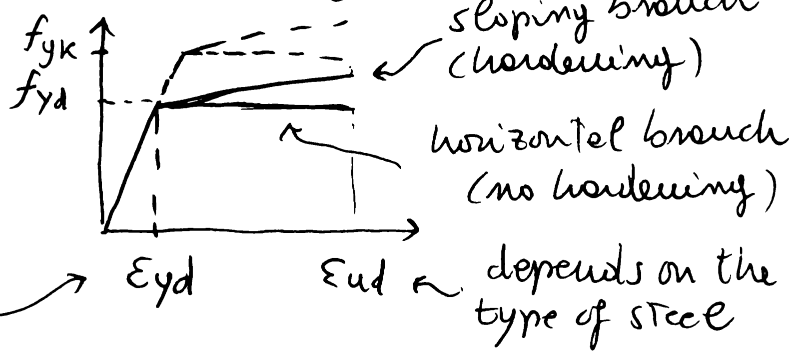
steel specified strength f_{yk} → yielding ~~strength~~ stress (1) / → specified proof stress (2)

Rebars are usually made of hot rolled steel

steel stress-strain laws (two options)

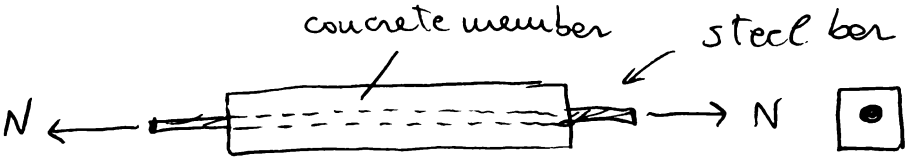
$$f_{yd} = \frac{f_{yk}}{\gamma_s} \qquad \gamma_s = 1.15$$

$$\varepsilon_{yd} = f_{yd}/E_s$$



⊙ BOND AND STRESS TRANSFER BETWEEN STEEL AND CONCRETE

Let consider a prism of concrete containing a central bar of steel (it would represent a rebar with the surrounding concrete), which is put in tension by an axial force N

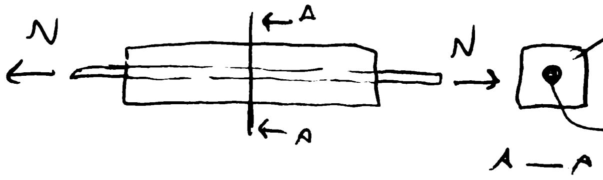


If cracking does not occur, concrete can be considered in the elastic stage

Perfect bond → same strain for steel and concrete

If you clamp the external portions of the steel bars, and apply the force to its ends, the steel outside will carry the entire force N. Inside the concrete, conversely, since both materials deform, the force should be partially carried by the steel, and partially carried by the concrete.

In other words, near the ends a stress transfer occurs, which reduces the force in the steel and increases the tension in the concrete.



A_c area of concrete

A_s area of steel

$A = A_c + A_s$ overall area

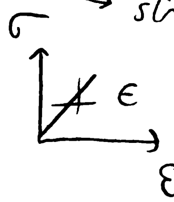
1) equilibrium $\longrightarrow$ $N = N_c + N_s$

N: overall force; N_c: carried by the concrete; N_s: carried by the steel

2) compatibility (congruence) $\varepsilon_c = \varepsilon_s$

ε_c: strain of concrete; ε_s: strain of steel

Linear elasticity assumption:



$$\frac{\sigma}{\varepsilon} = E \longrightarrow \sigma = E \cdot \varepsilon$$

1) $N = N_c + N_s \Rightarrow N = A_c \cdot \sigma_c + A_s \cdot \sigma_s \; \left(= A_c \cdot E_c \cdot \varepsilon_c + A_s E_s \varepsilon_s\right)$

σ_c: tension in concrete; σ_s: tension in steel

2) $\varepsilon_c = \varepsilon_s \Rightarrow \dfrac{\sigma_c}{E_c} = \dfrac{\sigma_s}{E_s} \longrightarrow \sigma_s = \left(\dfrac{E_s}{E_c}\right) \cdot \sigma_c \longrightarrow m = E_s / E_c$

$\sigma_s = m\,\sigma_c$

$$N = A_c\,\sigma_c + A_s \cdot \overbrace{m\,\sigma_c}^{\sigma_s} = \underbrace{(A_c + m A_s)}_{A_{id}} \cdot \sigma_c$$

A_{id} ideal area where steel counts m times

$$\sigma_c = \frac{N}{A_{id}}$$

$$N_c = A_c \sigma_c = \frac{A_c}{A_{id}} \cdot N$$

$$N_s = A_s \sigma_s = A_s \cdot m \sigma_c = \frac{m A_s}{A_{id}} \cdot N$$

here $\sigma_s = \dfrac{N}{A_s}$, because there is no concrete and therefore $\sigma_c = 0$

here $\sigma_c = N/A_{id}$ and $\sigma_s = m\sigma_c$

near the ends there is a stress transfer that reduces the tension in the steel from N/A_s to $m \dfrac{A_s}{A_{id}} \cdot N$, while the tension in concrete increases from 0 to σ_c

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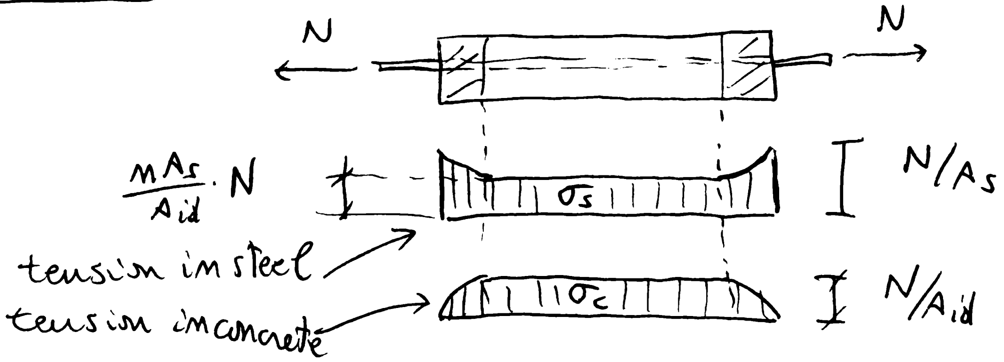
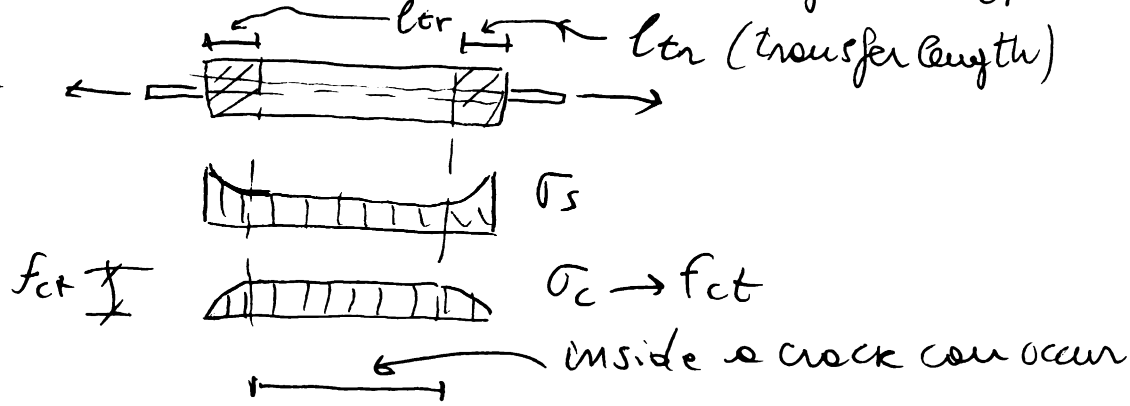


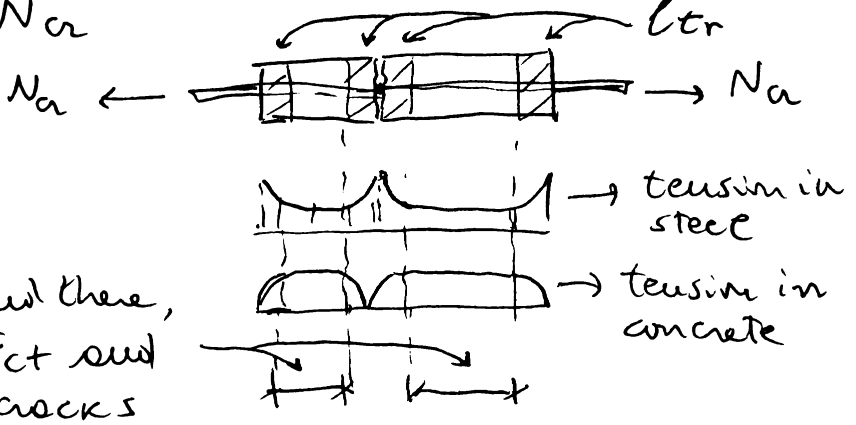
If we make N increase, σ_c will increase accordingly. At a certain moment, σ_c will equates f_{ct}, tensile strength of concrete, and a crack will occur inside the portion where $\sigma_c = f_{ct}$. The length required to transfer a stress to the concrete equal to f_{ct} is called transfer length ℓ_{tr}

Limit: $\sigma_c = f_{ct}$

$N = N_{cr}$



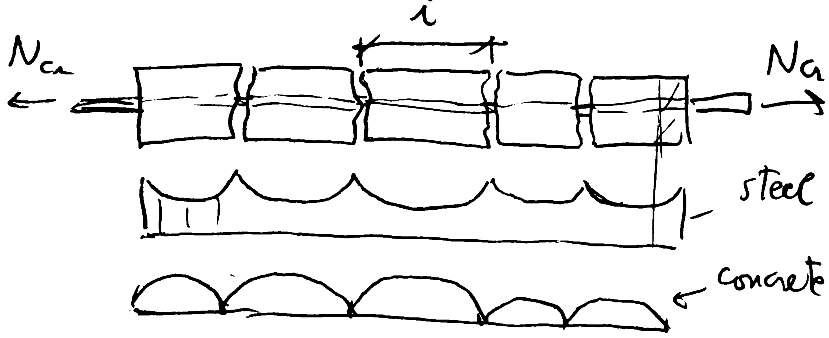
After the first crack occurs, N remains constant and equal to N_{cr}



Here and there, $\sigma_c = f_{ct}$ and other cracks can occur

At the crack: concrete is broken, so it carries no load, and steel carries the whole N_{cr} like at the exterior ends

After a complete cracking:



(i) → distance between two adjacent cracks

Let consider the range of variation for i

- Could i be lower than l_{tr} (transfer length)?

$l_{tr} \rightarrow$ length required to allow σ_c increasing from 0 to f_{ct}

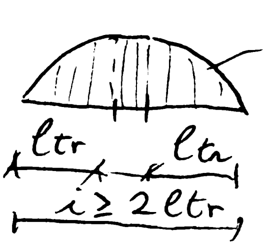
if $i < l_{tr} \Rightarrow$

$\leftarrow \sigma_c < f_{ct}$ therefore a crack could not occur

$\rightarrow$ therefore, $i \geq l_{tr}$: the distance between two adjacent cracks can be at least equal to the transfer length l_{tr}, otherwise σ_c cannot reach its maximum value f_{ct} (tensile strength of concrete)

- Can i be greater than $2 \cdot l_{tr}$?

if $i \geq 2 l_{tr} \Rightarrow$



$\Rightarrow$ there is at least one point (if $i = 2 l_{tr}$) where $\sigma_c = f_{ct}$, therefore a crack occurs, sooner or later, thus it is not possible

Therefore:

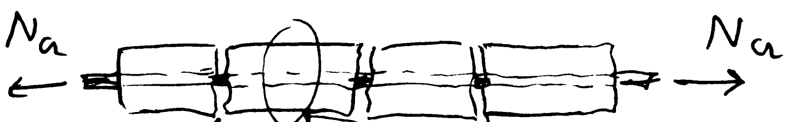
$$\boxed{l_{tr} \leq i < 2 \cdot l_{tr}}$$

- Let consider an average distance between cracks, i_{cr}

$i_{cr} = \alpha \cdot l_{tr}$, with $1 \leq \alpha < 2$

For simplicity, we can ~~write~~ consider i_{cr} proportional to the transfer length, even if we do not know the value of α (we just know that $\alpha \geq 1$ and $\alpha < 2$)

Between two subsequent cracks, there is a stress transfer between steel and concrete, and therefore a transfer of load.



Here, $N_{steel} = N_{cr}$

inside this portion, where concrete is not cracked,

$N_{cr} = N_c + N_s$

$N_s \rightarrow$ portion carried by steel

$N_c \rightarrow$ portion carried by concrete

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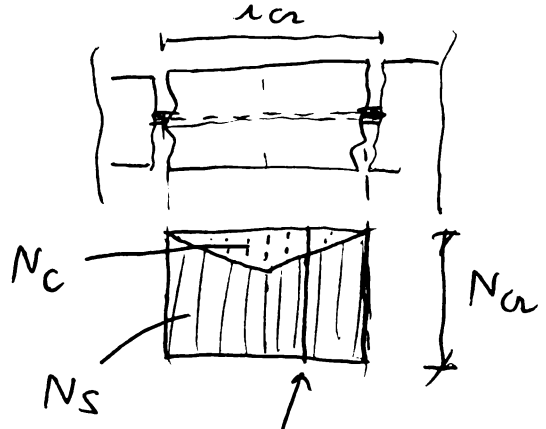


in every cross section,
$N_s + N_c = N_{cr}$

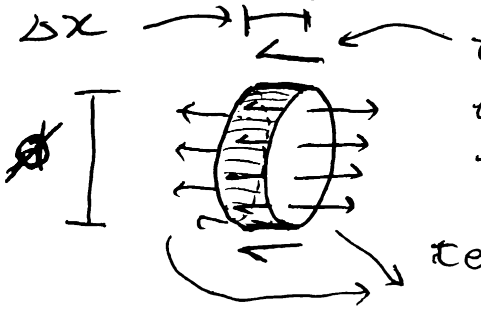
Of course, since inside the concrete portion the stress transfer is ongoing and not completely developed, the proportion between N_c and N_s is different in every point, although their distribution can be considered symmetric.

$N_s = N_{cr}$ at the two ends, while N_s is minimum in the centre, where N_c is maximum

Minimum value of force in the steel: $N_{s,min}$ (where N_c is max)
Maximum value of " " " " $N_{s,max} = N_{cr}$
Maximum value of force in the concrete: $N_{c,max}$

For simplicity, we consider a linear variation of force inside the element.
It is to be noted that, for the equilibrium of the cross section, the reduction of axial force in the steel equates the increment of tensile force in the concrete.

Let consider a small portion of steel inside the concrete:



tangential stresses τ_d acting along the lateral surface of the steel bar, at the interface with concrete

tensile stress in the steel bar

Being ϕ the diameter of the bar, the equilibrium of that small element of steel bar yields: $\Delta\sigma_s \cdot A_s = \tau_d \cdot A_{s,lateral}$

$$\underbrace{\Delta\sigma_s \cdot \underbrace{\frac{\phi^2}{4}\pi}_{\text{cross sectional area}}}_{\Delta N_s \text{ variation of the force in the steel (stress} \times \text{area)}} = \tau_d \cdot \underbrace{\pi\phi\Delta x}_{\text{lateral surface}}$$

(Δx is the length of the element)

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we got: $\Delta N_s = \tau_d \underbrace{\pi \phi}_{\text{perimeter}} \Delta x$

$$\frac{\Delta N_s}{\Delta x} = \tau_d \pi \phi$$

Since we assumed a linear variation of forces inside the reinforced concrete portion between two subsequent cracks, therefore we can consider τ_d constant, ~~since it is proportional~~

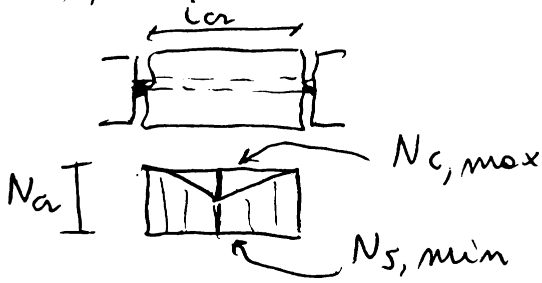
$$\frac{\Delta N_s}{\Delta x} \text{ linear} \longrightarrow \tau_d \text{ constant}$$

Moreover, we saw that $\Delta N_s = \Delta N_c$ in absolute values, since what steel looses, concrete gains.

$$\longrightarrow \frac{\Delta N_s}{\Delta x} = \frac{\Delta N_c}{\Delta x} \left(\begin{array}{c}\text{without}\\ \text{reference}\\ \text{to the sign}\end{array}\right) \Rightarrow \Delta x \to 0 \Rightarrow \frac{dN_c}{dx} = \tau_d \pi \phi$$

$$dN_c = \tau_d \pi \phi \cdot dx$$

Now, we want to calculate the maximum value of concrete force $N_{c,max}$ (in the middle), then we have to integrate dN_c from 0 to $i_{cr}/2$



$$N_{c,max} = \int_0^{i_{cr}/2} dN_c$$

$$= \int_0^{i_{cr}/2} \tau_d \pi \phi \, dx$$

$\pi \phi \longrightarrow$ constant (bar's perimeter)

$\tau_d \longrightarrow$ constant (linear variations of N_c, N_s)

$$N_{c,max} = \tau_d \pi \phi \int_0^{i_{cr}/2} dx = \frac{1}{2} i_{cr} \cdot \tau_d \pi \phi$$

Then, we want to calculate the average value of the tensile force in the steel between two cracks, $N_{s,avg}$

$$N_{s,avg} = \frac{N_{s,max} + N_{s,min}}{2} = N_{cr} - \frac{1}{2} N_{c,max}$$

linear variation!

$N_{s,max} = N_{cr}$

$N_{s,min} = N_{cr} - N_{c,max}$

calculate the average tension

$$\sigma_{s,avg} = \frac{N_{s,avg}}{A_s} = \frac{N_{cr}}{A_s} - \frac{i_{cr} \cdot \tau_d \pi \phi}{2 \cdot 2 \cdot \pi \phi^2 / 4} = \overline{\sigma_s} - \frac{\tau_d \cdot i_{cr}}{\phi}$$

let call it $\overline{\sigma_s}$

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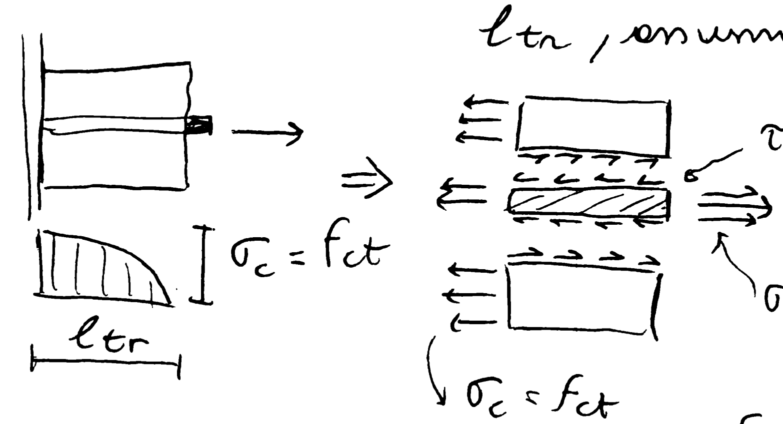
Let's calculate the transfer length l_{tr}, assuming a constant bond stress τ_d



τ_d mutually exchanged at the steel-concrete interface

Equilibrium of concrete only at a distance l_{tr}

$$\sigma_c A_c = \tau_d \pi \phi \cdot l_{tr}$$

σ_c: f_{ct}!

τ_d: constant along the length

Thus $l_{tr} = \dfrac{f_{ct} A_c}{\tau_d \pi \phi} = \dfrac{f_{ct}\, \phi \cdot A_c}{4 \tau_d \cdot A_s}$

Let's rewrite $\pi\phi$: $A_s = \pi \dfrac{\phi^2}{4}$

$\Rightarrow \pi \phi = 4 A_s / \phi$

Be $\rho = \dfrac{A_s}{A_c}$ (rho) $\Rightarrow l_{tr} = \dfrac{\phi f_{ct}}{4 \tau_d \cdot \rho}$

Therefore, the average stress in steel is $\sigma_{s,avg} = \overline{\sigma_s} - \dfrac{\cancel{\tau_d}}{\cancel{\phi}} \cdot \underbrace{\alpha \cdot \dfrac{\cancel{\phi} f_{ct}}{4 \cancel{\tau_d} \rho}}_{i_a = \alpha l_{tr}}$

$$\sigma_{s,avg} = \overline{\sigma_s} - \alpha \frac{f_{ct}}{4\rho}$$

$\overline{\sigma_s}$: $\dfrac{N_a}{A_s}$

The average strain in the steel is given by $\sigma_{s,avg} / E_s = \varepsilon_{s,avg}$

Now, we can observe that, since the concrete between two cracks is in its elastic range (small strains, and the steel too, because for the perfect bond $\varepsilon_s = \varepsilon_c$), all the deformations are concentrated at the cracks, therefore the average crack width w_a can be calculated as $w_a = \underbrace{i_a}_{\text{average distance between two cracks}} \cdot \underbrace{\varepsilon_{s,avg}}_{\text{average strain of the steel between two cracks}}$

being $i_a = \alpha\, l_{tr}$ we get:

$$w_a = \alpha \cdot \frac{\phi f_{ct}}{4 \tau_d \rho} \cdot \frac{1}{E_s} \cdot \left(\overline{\sigma_s} - \alpha \frac{f_{ct}}{4\rho} \right)$$

w_a: average opening of a crack (displacements concentrated ~~here~~ here)

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Looking at the average width of a crack w_{cr}:

$$w_{cr} = \frac{\alpha f_{ct} \cdot \phi}{4 E_s \tau_d \rho} \left(\bar{\sigma}_s - \frac{\alpha f_{ct}}{4 \rho} \right)$$

The crack width decreases (single crack!)

- when the working stress of steel $\bar{\sigma}_s = N/A_s$ decreases
- when the diameter of the steel bar decreases (smaller diameters reduce the crack width)
- when the bond stress τ_d improves
- when the percentage of reinforcement $\rho = A_s/A_c$ increases
- when the tensile strength of concrete ~~improves~~ decreases

Now, let consider the overall crack width Σw_{cr}, along the length L of the member.

Within L, you can find, in average, n_{cr} cracks, being

$$n_{cr} = L/i_{cr} = \frac{\text{overall length of the member}}{\text{average distance between to cracks}}$$

$$\Sigma w_{cr} = n_{cr} \cdot w_{cr} = \frac{L}{\underbrace{\cancel{\alpha \ell_{tr}}}_{i_{cr}}} \cdot \underbrace{\cancel{\alpha \ell_{tr}}}_{i_{cr}} \cdot \underbrace{\frac{1}{E_s} \left(\bar{\sigma}_s - \frac{\alpha f_{ct}}{4 \rho} \right)}_{\varepsilon_{s,avg}}$$

$$\downarrow$$

$$\Sigma w_{cr} = \frac{L}{E_s} \left(\bar{\sigma}_s - \frac{\alpha f_{ct}}{4 \rho} \right)$$

The overall crack width does not depend anymore upon the diameter ϕ of the bar and upon the bond stress, but still depends on the working stress of steel $\bar{\sigma}_s$, the reinforcement percentage $\rho = A_s/A_c$, and the concrete tensile strength f_{ct}.