LECTURE 4

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Yaoundé, 21/11/2014

GENERAL ANALYSIS OF RECTANGULAR CROSS SECTIONS

Premise: When dealing with this topic, as usual we assume that:

1 _ the contribution of concrete in tension (although important for other reasons) ~~con~~ to the capacity of the section can be disregarded.

2 _ Materials stress-strain laws are chosen among the proposed ones, already seen (for concrete: parabola + plateau; linear + plateau; stress-block; for steel: linear + plateau, linear + hardening branch)

3 _ There is a perfect bonding between concrete and reinforcing steel.

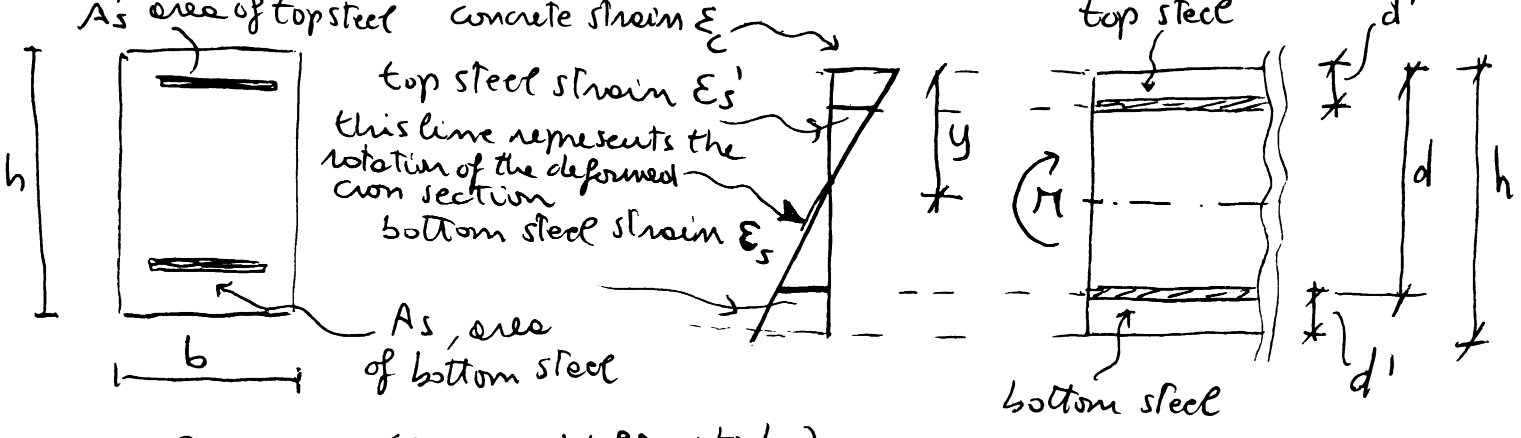
4 _ Plane sections remain plane, also after deformation..

As a general rule, when designing a cross section, the aim should be to have a ductile failure mode, instead of a brittle one.

Ductile failures occur with great deformation of steel, so they are apparent, evident; redundant structures allow for a redistribution of internal forces, provided that sections have a good rotational capacity. Moreover, ductile failures imply a great energy absorption / dissipation.

Conversely, brittle failures are dangerous, abrupt and not evident, and do not allow for a redistribution of forces along the structural members of a redundant structure.

DEFORMATION OF A GENERIC RECTANGULAR CROSS SECTION



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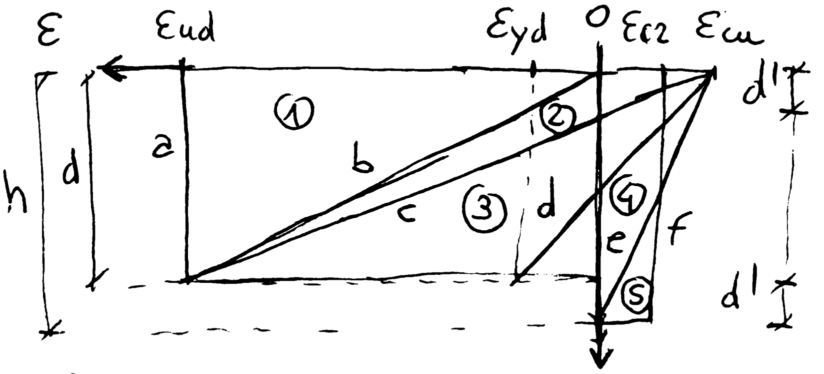
All the possible configurations that a generic cross section can assume are limited by the properties of materials (concrete and steel).

The most general situation is plotted in the following.



There are 6 critical lines (a, b, c, d, e and f) that separate 5 fields (1, 2, 3, 4 and 5)

By assumption, we ~~assu~~ consider that steel has the same cover (bottom and top)

h, b → depth and width of the cross section

d → effective depth of the cross section (distance from the barycentre of steel and the opposite edge of the section

d' → cover (distance from the barycentre of steel to the ~~clo~~ nearer edge of the section).

ε_{ud} → maximum design strain of steel in tension (design value)

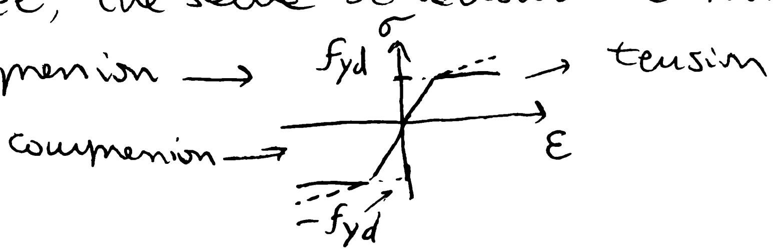
ε_{yd} → yielding strain of steel in tension (design value)

ε_{cu} → maximum strain in compression for concrete (design value)

ε_{c2} → maximum strain in compression when the section is uniformly compressed (this limitation, which could be taken, alternatively, as ε_{c3}) is considered in order to ensure a sufficient reserve of deformation for the section entirely compressed) (design value)

y → neutral axis depth, taken as: $y = \xi \cdot d$

Note that, for steel, the same behaviour is assumed for both tension and compression →



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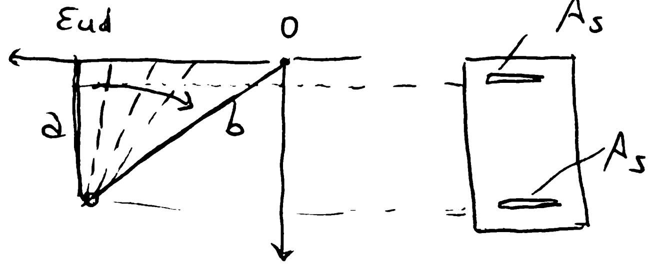
Line a corresponds to $y = -\infty \Rightarrow \xi_a = -\infty$ $\quad [y = \xi d]$

Line b corresponds to $y = 0 \Rightarrow \xi_b = 0$

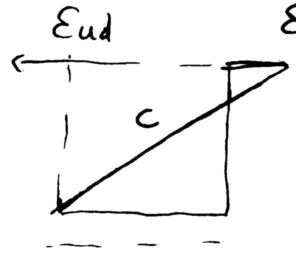
Field ① comprises all the situations between $-\infty$ and 0

i.e. $-\infty < \xi \leq 0$,

where the bottom steel has reached its maximum strain (ε_{ud})



Line c



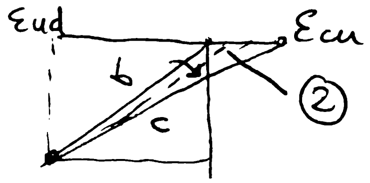
$y_c = \xi_c \cdot d$ $\qquad \dfrac{\varepsilon_{cu}}{\xi_c \cdot d} = \dfrac{\varepsilon_{ud} + \varepsilon_{cu}}{d}$

relations between similar triangles

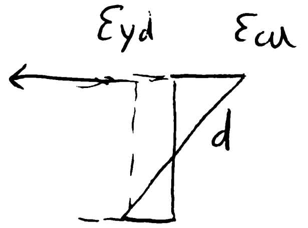
$$\rightarrow \xi_c = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{ud}}$$

Field ② comprises all the situations where the bottom steel has reached its maximum strain ε_{ud}, and the concrete strain varies from 0 to its maximum value ε_{cu}

$$0 < \xi \leq \xi_c = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{ud}}$$



Line d

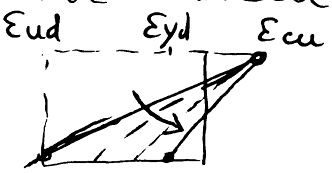


$y_d = \xi_d \cdot d$ $\qquad \dfrac{\varepsilon_{cu}}{\xi_d \cdot d} = \dfrac{\varepsilon_{cu} + \varepsilon_{yd}}{d}$

$$\rightarrow \xi_d = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}}$$

Field ③ comprises all the situations where the bottom steel has a strain included between its ultimate strain ε_{ud} and its yielding strain ε_{yd}, while concrete has reached its maximum compressive strain ε_{cu}

$$\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{ud}} < \xi \leq \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = \xi_d$$



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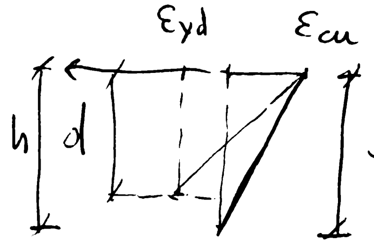
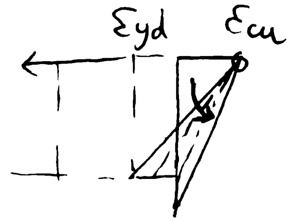
Line e



$y_e = h = \xi_e \cdot d \rightarrow \xi_e = \frac{h}{d}$

Field (4) comprises all the situations where concrete has reached its maximum strain ε_{cu}, and the section becomes entirely compressed (therefore also the concrete of the bottom cover can react)

$$\frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{yd}} < \xi \leq \frac{h}{d} = \xi_e$$



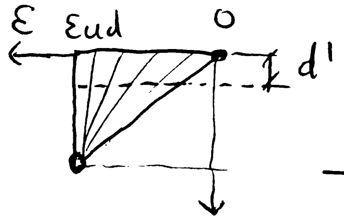
Line f the section is uniformly compressed, therefore $y_f \rightarrow +\infty$, like ξ_f

Field (5) comprises all the situations where the section is entirely compressed (but the compressive strain cannot be greater than ε_{c2} ~~and ε_{c2}~~ ~~at the same time~~) ~~on one side and equal~~ in all the section)

$$\frac{h}{d} < \xi < +\infty$$

Let consider the possible combination of internal forces related to each field.

Field (1)

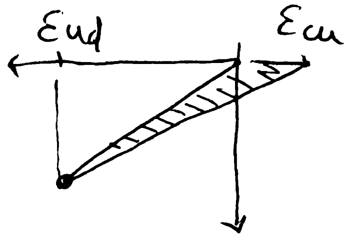


All the section is in tension.

Possible ~~comb.~~ internal forces:

- pure tension $N>0$, $M=0$ (if $A_s = A'_s \Rightarrow T_s = T'_s$)
- combined tension and bending $N>0$, $M \neq 0$ (if $T_s \neq T'_s$)

Field (2)



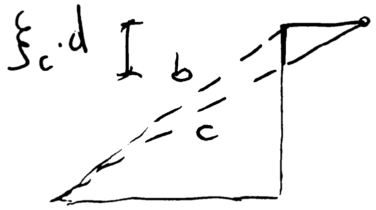
The neutral axis crosses an internal point of the section, therefore pure tension is not possible.

In theory, you could have:

- combined bending and tension, $N>0$, $M \neq 0$
- combined bending and compression, $N<0$, $M \neq 0$
- pure bending, $N=0$, $M \neq 0$

In practice, let's have a look deeper into this field

Let consider a steel 450C $\rightarrow f_{yk} = 450$ MPa



$\varepsilon_{ud} = 0.9\, \varepsilon_{uk} = 0.9 \cdot 75‰ = 67.5‰$

$\varepsilon_{cu} = 3.5‰$

$$\xi_c = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{ud}} = 0.049 \rightarrow y \cong 5\%\, d$$

a very small portion of the cross section is compressed, therefore the compressive force withstood by concrete is low

Consider the strain of the top steel: ε_s'

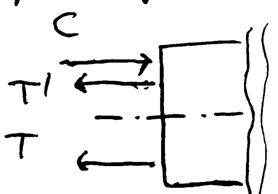
5%d is a value that can be adopted, in practice, for the cover. Be $\delta = d'/d \Rightarrow$ usual ranges for δ are 0.05 - 0.15

Therefore, the top steel strain is close to zero or is positive (i.e. also the top steel is in tension like the bottom one).

To have simple bending, or combined bending and compression,

$C \geq T + T' \longrightarrow$ tensile forces for bottom and top steel

$\hookrightarrow$ compression force in the concrete



The minimum area of tensile reinforcement that has to be provided, $A_{s,min}$, according to Eurocode 2 is:

$$A_{s,min} = 0.26 \frac{f_{ctm}}{f_{yk}} b_t \cdot d$$

($f_{ctm} = 0.3 \sqrt[3]{f_{ck}^2}$ $\rightarrow$ average tensile strength of concrete)

b_t $\rightarrow$ width of the concrete section measured at the tension side (rectangular $\rightarrow b_t = b$)

$$= 0.26 \cdot 0.3 \sqrt[3]{f_{ck}^2}\, bd / f_{yk}$$

$f_{ck} = \frac{1.5}{0.85} f_{cd}$

$f_{yk} = 1.15\, f_{yd}$

$$= 0.26 \cdot 0.3 \sqrt[3]{\left(\frac{1.5}{0.85}\right)^2 f_{cd}^2} \cdot bd / 1.15\, f_{yd}$$

$$\cong 0.099 \sqrt[3]{f_{cd}^2}\, bd / f_{yd}$$

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$$C = \underbrace{0.8 \cdot y_c \cdot b \cdot f_{cd}}_{\text{a stress-block is adopted}}$$ compression force for the maximum area ($\rightarrow$ max depth of the neutral axis)

$$C = 0.8\, \xi_c\, d\, b\, f_{cd} \geq T + T'$$

T → bottom steel

T' → top steel

$T = f_{yd} \cdot A_s \longrightarrow$ minimum tensile force on the bottom steel for A_s equal to $A_{s,min}$

$$T_{min} = f_{yd} \cdot A_{s,min} \cong \cancel{f_{yd}} \cdot 0.099 \sqrt[3]{f_{cd}^2}\, bd / \cancel{f_{yd}}$$

$$\rightarrow 0.8\, \xi_c\, \cancel{bd}\, f_{cd} \geq 0.099 \sqrt[3]{f_{cd}^2}\, \cancel{bd} + T'/bd$$

$$\xi_c \geq \frac{0.099}{0.8} \frac{\sqrt[3]{f_{cd}^2}}{f_{cd}} + T'/0.8\, bd\, f_{cd}$$

$$\xi_c \geq 0.124 \frac{1}{\sqrt[3]{f_{cd}}} + T'/0.8\, bd\, f_{cd}$$

Let's consider, for example, a concrete $C30/37 \Rightarrow f_{cd} = 17\,MPa$

$$\rightarrow \underbrace{\xi_c \geq \frac{0.124}{\sqrt[3]{17}}}_{0.048\, \approx\, \xi_c} + T'/0.8\, bd\, f_{cd}$$

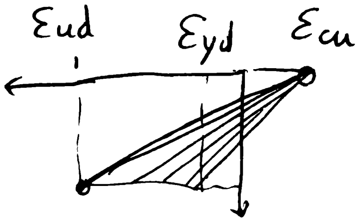
$0.048 \approx \xi_c$, but without considering the top steel

Conclusion: in most cases, having simple bending or combined bending and compression in the field (2) is rarely possible (only if you have a very low amount of steel and a very strong concrete).
Moreover, it should be considered that, for ordinary beams, the area of compressed concrete is very small and, generally, involves more or less the meat cover plus something more, and this, however, should be avoided

In most cases, field (2) $\longrightarrow$ combined bending and tension

$N > 0,\ M \neq 0$

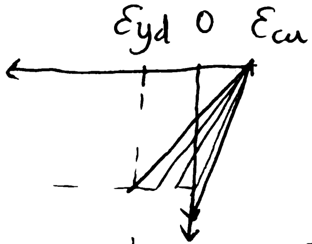
Field (3)



the neutral axis intersect the cross section, therefore you can have
- combined bending and tension: ~~compression~~
  $N>0$, $M\neq 0$
- simple bending $N=0$, $M\neq 0$
- combined bending and compression
  $N<0$, $M\neq 0$

For bending, field (3) is the field of ductile failures (crushing of concrete, steel yielded)

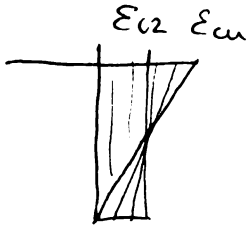
Field (4)



As the previous fields, the neutral axis ~~comes~~ intersects the cross section and you can have:
- Combined bending and tension
  $N>0$, $M\neq 0$
- simple bending $N=0$, $M\neq 0$
- combined bending and compression
  $N<0$, $M\neq 0$

For bending, field (4) is the field of brittle failures (crushing of concrete with steel in the elastic range)

Field (5)



All the section is under compression, therefore you can have:
- combined bending and compression
  $N<0$, $M\neq 0$
- Simple compression $N<0$, $M=0$

SUMMARY

FIELD	NEUTRAL AXIS POSITION	CONCRETE STRAIN	STEEL STRAIN	POSSIBLE INTERNAL FORCES	FAILURE
(1)	$-\infty<\xi\leq 0$	$\varepsilon_c>0$	ε_{ud}	$N>0$. $M=0$ $N>0$ $M\neq 0$	steel in tension
(2)	$0<\xi\leq \dfrac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{ud}}$	$0\div\varepsilon_{cu}$	ε_{ud}	$N>0$, $M\neq 0$ $N=0$, $M\neq 0$ $N<0$, $M\neq 0$	steel in tension
(3)	$\dfrac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{ud}}<\xi\leq\dfrac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{yd}}$	ε_{cu}	$\varepsilon_{yd}\div\varepsilon_{ud}$	like (2)	crushing of concrete
(4)	$\dfrac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_{yd}}<\xi\leq h/d$	ε_{cu}	$<\varepsilon_{yd}$	like (2)	crushing of concrete
(5)	$h/d<\xi<+\infty$	$\varepsilon_{c2}\div\varepsilon_{cu}$	<0	$N<0$, $M=0$ $N<0$, $M\neq 0$	crushing of concrete

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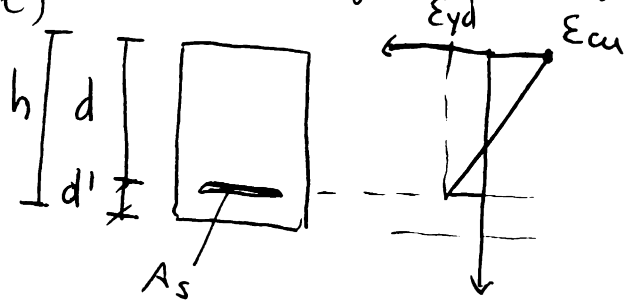
⊙ THE BALANCED SECTION

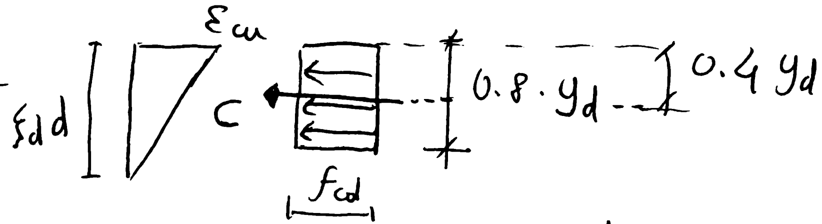
It is the section where failure occurs due to concrete crushing, and steel reaches the yielding strain $\rightarrow$ ε_{cu} ; ε_{yd}

Let's consider simple bending and single reinforcement (only bottom steel)



$$\xi = \xi_d = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}}$$

Compression in the concrete (adopting a stress-block)



$C = 0.8\, \xi_d \cdot d \cdot b \cdot f_{cd}$ $\rightarrow$ located at $0.4\, \xi_d \cdot d$ from the top edge

Tension in the (bottom) steel, that is yielding

$$T = f_{yd} \cdot A_s$$

Simple bending $\rightarrow$ $N = 0, M \neq 0 \Rightarrow C = T \rightarrow 0.8\, \xi_d\, b d f_{cd} = f_{yd} A_s$

Be $\rho = \dfrac{A_s}{b \cdot d} \Rightarrow A_s = \rho\, b d$

Since ξ_d depends only on the properties of steel and concrete (ε_{cu} and ε_{yd}), there is only one amount of steel that equilibrates the force in the concrete $\rightarrow$ ρ_{bal}

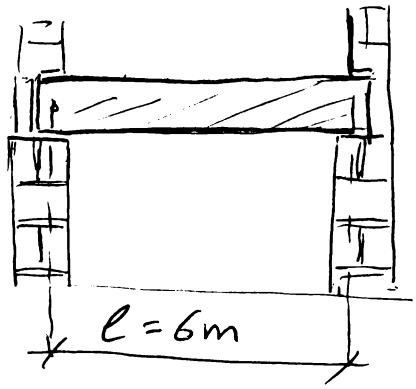
$$0.8\, \xi_d \cdot \cancel{bd}\, f_{cd} = f_{yd} \cdot \rho_{bal} \cdot \cancel{b \cdot d} \Rightarrow \rho_{bal} = 0.8\, \xi_d \cdot f_{cd}/f_{yd}$$

If $\rho > \rho_{bal} \Rightarrow$ bending + tension $(C < T)$

If $\rho < \rho_{bal} \Rightarrow$ bending + compression $(C > T)$

$\rho_{bal} \rightarrow$ max steel reinforcement, in bending, to have a ductile failure. Otherwise, failure in simple bending will occur with steel in the elastic range, since a wider area of compressed concrete $(\xi > \xi_d)$ is required to equilibrate the tension force in the steel

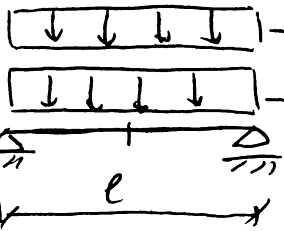
EXAMPLE OF DESIGN AND ASSESSMENT (SIMPLE BENDING)



simple supported beam, with a design span $l = 6m$.

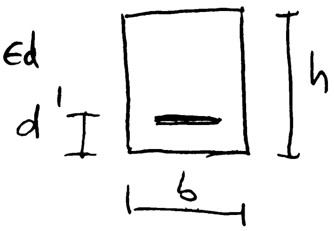
Permanent ~~wa~~ load $g_K = 19$ kN/m

variable load $q_K = 10$ kN/m



$1.50\, q_K$, $1.35\, g_K$ } design loads for the middle section

$$M(l/2) = \frac{1}{8}(1.5 q_K + 1.35 g_K)\, l^2 = \frac{1.5 \cdot 10 + 1.35 \cdot 19}{8} \cdot 36 = 183 \text{ kN}\cdot\text{m} = M_{Ed}$$

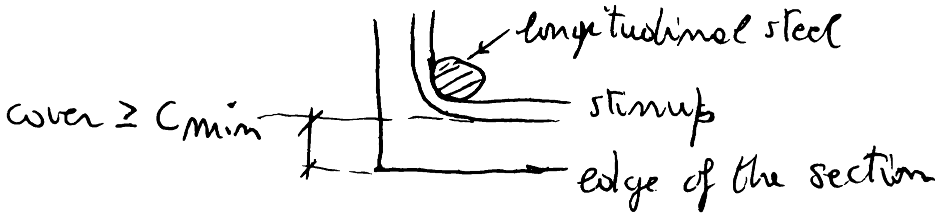


Choose a reasonable dimension for the cross section, i.e. $b = 30$ cm, $h = 50$ cm

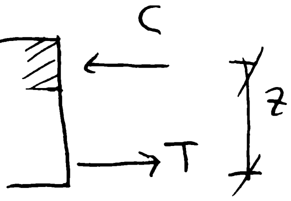
assign a practical value of $\delta = d'/d$, for example $\delta = 0.1$

$\Rightarrow d' = 10\% \cdot d = 4.5$ cm $\Rightarrow d' = 5$ cm

since h is not large, you should improve δ to have a reasonable cover



Predesign formulation: z = lever arm of internal forces $\cong 0.9d$



$z \cong 0.9d$ as first approximation

$$T \cdot z = C \cdot z = M \Rightarrow M_{Ed} = T \cdot z$$

$M_{Ed} = 0.9d \cdot A_{s,min} \cdot f_{yd}$ ← find the minimum amount of steel

$$A_{s,min} = \frac{183 \text{ kN}\cdot\text{m}}{0.9 \cdot 450\text{mm} \cdot 391 \text{ MPa}} \rightarrow 1156 \text{ mm}^2$$

B450C → $f_{yK} = 450$, $f_{yd} = 391$ MPa

$1\phi16 \rightarrow 201 \text{ mm}^2 \rightarrow 6$
$1\phi20 \rightarrow 314 \text{ mm}^2 \rightarrow 4$ →
$1\phi22 \rightarrow 380 \text{ mm}^2 \rightarrow 3$
$1\phi24 \rightarrow 452 \text{ mm}^2 \rightarrow 3$

let's choose $4\phi20 \rightarrow A_s = 1256 \text{ mm}^2$

$\rho_{s,l}$ = [concrete C25/30 → $f_{cd} = \frac{0.85 \cdot 25}{1.5} = 14$ MPa]

$= 0.8 \cdot \frac{3.5}{3.5 + 1.96} \cdot 14/391 = 0.0184 \rightarrow 2484 \text{ mm}^2$

Ductile failure verification →

$$\xi = \frac{A_s f_{yd}}{0.8\, b d f_{cd}} = 0.325 < \xi_d \Rightarrow \text{field } 3$$

$$M_{Rd} = f_{yd} A_s \cdot (d - \xi d \cdot 0.4) = 192 \text{ kNm}$$

$M_{Rd} > M_{Ed}$ ⇐ $192 > 183$

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