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RECAP if the column is short, second order effects are negligible, and the design or assessment can be performed with reference to the cross-sectional capacity, and this is enough.

If the column is slender, second order effects (i.e. those related to the deflection of the member, and to the sway of the frame) must be taken into account, because they can dramatically reduce the column's capacity. In other words, the analysis of the cross section is not enough to evaluate the resistance under combined bending moment and compression axial force, since the global behavior of the member must be taken into account.

We introduced the quantity $\lambda = l_0 / i$, called slenderness, and we said that a column is short if $\lambda < \lambda_{lim}$, and is slender if $\lambda \geq \lambda_{lim}$, being λ_{lim} a proper limit value.

Now, we can define how to calculate λ_{lim}, since we are able to evaluate the effective length l_0 and the radius of gyration i (thus, we can get λ)

CALCULATION OF λ_{lim}

$$\lambda_{lim} = 20 \cdot A \cdot B \cdot C \cdot 1/\sqrt{n}$$

Assumption: end 2 is the end with the greater value of bending moment in absolute value:

$$|M_2| > |M_1|$$

$$A = \frac{1}{1 + 0.2 \cdot \varphi_{ef}}$$

φ_{ef} = effective creep ratio $= \varphi(\infty, t_0) \cdot \dfrac{M_{0Eqp}}{M_{0Ed}}$

$\varphi(\infty, t_0)$ = final creep coefficient

M_{0Eqp} = first order moment calculated with the quasi-permanent combination

M_{0Ed} = first order moment for ULS

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 The final creep coefficient is calculated using a couple of charts provided by EC2. It depends upon the class of cement (R, S or N) and the strength class of concrete. Two different situations of environmental conditions are provided: inside and outside conditions.

EXAMPLE calculation of $\boxed{\varphi(\infty, t_0)}$

- You have to know t_0 = ~~the~~ age of the concrete at time of loading, in days

  t_0 could be, for example, 7 days or 28 days, accordingly to the construction schedule.

- You have to know the class of concrete, which depends upon the type of cement:

  CLASS R : for cement of strength classes CEM 42.5 R, CEM 52.5 N and CEM 52.5 R (written on the package!)

  Class N : for cement of strength classes CEM 32.5 R, CEM 42.5 N (written on the package)

  CLASS S : for cement of strength class CEM 32.5 N (written on the package!)

- You have to know the strength class of concrete.

- You have to compute the perimeter exposed to drying of your member u, which could be, for isolated members, the outer perimeter of the cross section, or however the size of the part that is exposed to air.

  Steps: calculate the notional size $h_0 = 2 \cdot A_c / u$, with A_c area of the concrete section

  ~~...~~ Choose the proper condition (inside - 50% of Relative Humidity - or outside - 80% of R.H.), i.e. the proper couple of charts.

  draw a straight* line in correspondence to t_0, until it crosses the right curve related to the class of concrete (S, N, R).

* horizontal

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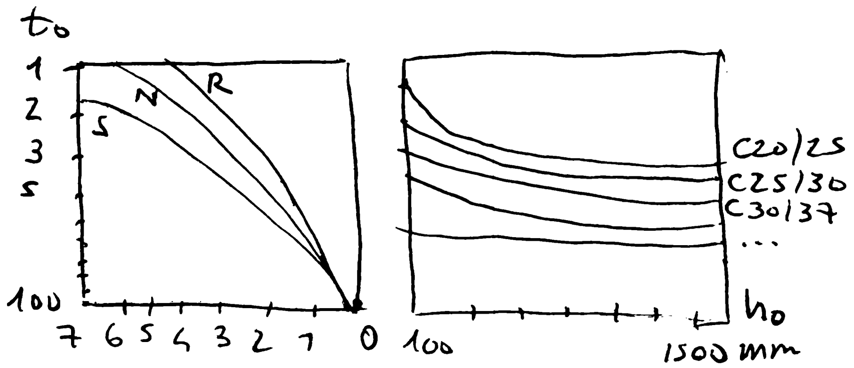
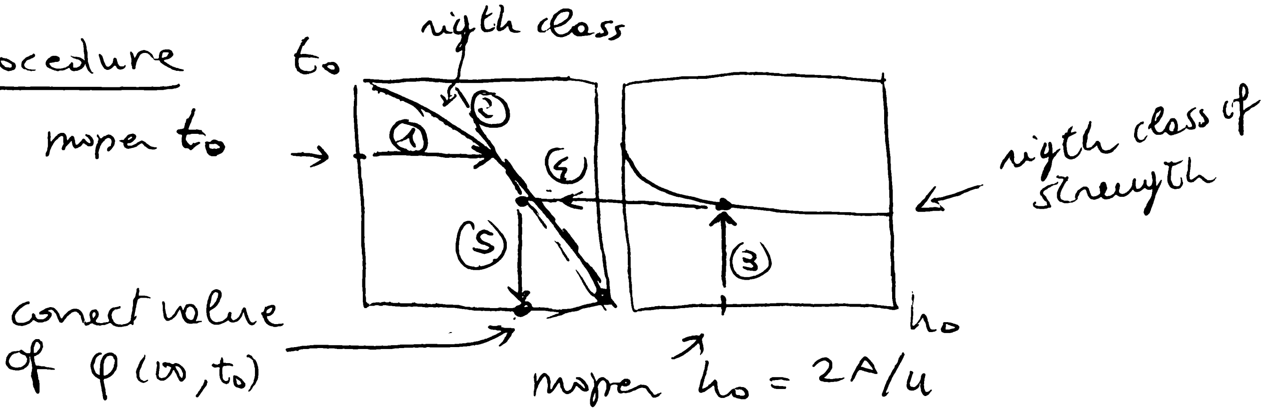
Plot a line that connects that point with the bottom-right corner.
Enter the second chart (on the rigth) with h_0, and draw a straigth vertical line until it crosses the proper strength class of concrete.
Connect that intersection with a horizontal line that goes to the left chart, until it crosses the previously plotted sloping line.
Go down and read $\varphi(\infty, t_0)$



Procedure



For the calculation of A, we need M_{0Eqp}, which is the first order moment (i.e. the value of bending moment calculated using a linear elastic analysis refered to the initial undeformed shape of the structure) calculated using the combination of loads called "quasi-permanent", related to serviceability limit states.
Converseley, M_{0Ed} is the design value of bending moment calculated using a first-order analysis but with the proper combination for ultimate limit states (i.e. it is the value of M given by the analysis)

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total steel area

$$\text{(B)} = \sqrt{1+2\omega} \qquad \text{with} \qquad \omega = \frac{A_s\, f_{yd}}{A_c\, f_{cd}}$$

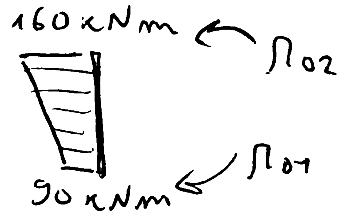
A_s = total area of the longitudinal steel rebars (top + bottom)
f_{yd} = design yielding strength of steel
f_{cd} = design compressive strength of ~~steel~~ concrete
A_c = area of concrete

$$\text{(C)} = 1.7 - r_m \qquad \text{with} \qquad r_m = \frac{M_{01}}{M_{02}}$$

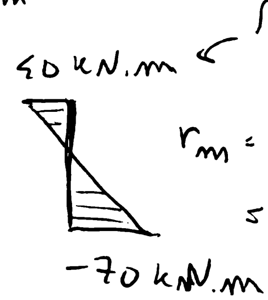
NOTE $[0.7 \le C \le 2.7]$

M_{01} and M_{02} are the first order moments at end 1 and 2, respectively, with $|M_{02}| > |M_{01}| \Rightarrow |r_m| \le 1$!
If M_{01} and M_{02} produce "tension" on the same side, $r_m > 0$, otherwise they have the opposite sign and $r_m < 0$

example



$$r_m = \frac{90}{160} = 0.563$$



$$r_m = -\frac{40}{70} = -0.571$$

$$\text{(n)} = \frac{N_{Ed}}{A_c \cdot f_{cd}}$$

N_{Ed} = design value of axial force
A_c = cross sectional area of concrete
f_{cd} = design strength of concrete

n is the relative (adimensional) axial force!

NOTE
- If the effective creep ratio φ_{ef} is not known, you may use $A = 0.7$
- If the mechanical percentage of reinforcement ω is not known, you may use $B = 1.1$
- If r_m is not known, C may be taken as 0.7
- For unbraced members in general, $r_m = 1 \Rightarrow C = 0.7$
- For braced members in which the first order moments arises only due to imperfections or lateral loading, $r_m = 1 \Rightarrow C = 0.7$

- approximations for λ_{lim}

1) Unbraced members; Braced members with M due only to imperfections or lateral loading

$$\lambda_{lim} \cong 20 \cdot \overset{A}{0.7} \cdot \overset{B}{1.1} \cdot \overset{C}{0.7} \Big/ \sqrt{N_{Ed}/A_c f_{cd}} \cong 10.8 \Big/ \sqrt{N_{Ed}/A_c f_{cd}}$$

2) Braced members with $M_{02} \cong M_{01}$, single curvature (they produce "tension" on the same side)

$$\lambda_{lim} \cong 20 \cdot \overset{A}{0.7} \cdot \overset{B}{1.1} \cdot \overset{C}{0.7} \Big/ \sqrt{N_{Ed}/A_c f_{cd}} \cong 10.8 \Big/ \sqrt{N_{Ed}/A_c f_{cd}}$$

3) Braced members with $|M_{02}| \cong |M_{01}|$, but double curvature $\rightarrow C = 1.7 - (-1)$

$$\lambda_{lim} \cong 20 \cdot \overset{A}{0.7} \cdot \overset{B}{1.1} \cdot \overset{C}{2.7} \Big/ \sqrt{N_{Ed}/A_c f_{cd}} \cong 41.6 \Big/ \sqrt{N_{Ed}/A_c f_{cd}}$$

4) Alternative for braced members: $r_m = 0 \rightarrow C = 1.7$

$$\lambda_{lim} \cong 26.2 \Big/ \sqrt{N_{Ed}/A_c f_{cd}}$$

HOW TO DEAL WITH SLENDER COLUMNS (i.e. $\lambda \geq \lambda_{lim}$)?

- The general method consist in performing from the beginning a non-linear analysis, which accounts for non-linear relations for each material and for the inclusion of the effects of deformations on the distribution and amount of internal forces.
- Another option is to perform an elastic analysis that includes second order effects.
- From a practical point of view, however, some simplified methods are available, which consists in procedures to calculate an increased value of design bending moment M to be used in the verification of a slender column: in other words, a slender column can be designed as a short one, provided that the design value of bending moment has been properly increased

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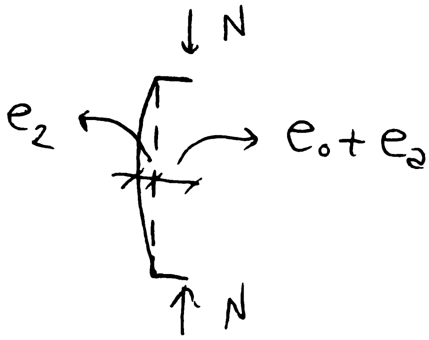
THE "NOMINAL CURVATURE" METHOD for slender columns



It consists in a simplified way to estimate the maximum second order eccentricity e_2, to be added to the first order and the "accidental" one.

Initial situation: from your first-order analyses, you get a certain value of M_0 and axial force N_{Ed}. M_0 (bending moment related to the linear analysis) has to be increased by a quantity that accounts for imperfections during construction. It is more practical to refer to an axial force placed at a certain distance e (eccentricity)), instead of referring to a couple (N, M) calculated in correspondence to the longitudinal axis of your element.

Of course, $N \cdot e = M \Rightarrow e = M/N$.

So we get N_{Ed} and $M_{0Ed} = M_0 + N_{Ed} \cdot e_a$ as first order design forces (thus $M_{0Ed} = N_{Ed} \cdot e_0 + N_{Ed} \cdot e_a = N_{Ed}(e_0 + e_a)$), being e_0 the first-order eccentricity and e_a the "accidental" eccentricity related to imperfections.

Then: if the column is short ($\lambda < \lambda_{lim}$), you can design it with reference to the cross sectional capacity, using the values of N_{Ed} and M_{0Ed} as design forces.

Conversely, if the column is slender ($\lambda \geq \lambda_{lim}$), you have to consider (N_{Ed} is kept constant, not increased) that your design bending moment has to be increased by adding a quantity that accounts for the second-order effects. In other words, your design bending moment M_{Ed} will be equal to $N_{Ed}(e_0 + e_a) + \underbrace{N_{Ed} \cdot e_2}_{M_2 \text{ related to second order effects.}}$

Tipically, if the column is not laterally loaded, you will have a linear (at least constant) distribution of bending moment, if the column's ends are subjected to axial force and bending moment.

You could have different first order moments M_{01} and M_{02} at the ends of the column.

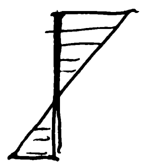
In this case, provided that $|M_{02}| > |M_{01}|$ (thus you will call "end 1" the end with the lower bending moment, in absolute value, and the other "end 2"), you may replace your maximum value of bending moment (M_{02}) with a reduced value, called equivalent moment M_{0e}.

$M_{0e} = 0.6\, M_{02} + 0.4\, M_{01}$, ~~with the~~ with the restraint that

$M_{0e} \geq 0.4\, M_{02}$

EXAMPLE

70 kN.m



end 2!

end 1!

10 kN.m

NOTE if M_{01} and M_{02} give tension on the same side they have the same sign, otherwise one is greater than 0, the other lower than 0.

Instead of designing the column for $|M_{02}| = 70$ kN·m, you may calculate an equivalent value: $M_{0e} = 0.6\, M_{02} + 0.4\, M_{01}$

$= 0.6 \cdot 70 + 0.4\,(-10)$

$= 42 - 4 = 38$ kN·m $> 0.4 \cdot 70 = 28$

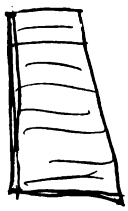
38 instead of 70 (less demanding)

EXAMPLE

70 kN·m

end 1!

end 2!



120 kN·m

$M_{0e} = (0.6 \cdot 120 + 0.4 \cdot 70)$ kN·m

$= 72 + 28 = 100$ kNm > 40 kNm

100 instead of 120 → less demanding

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Page 8 The eccentricity to cover imperfections, e_a, may be taken equal to $l_0/400$ (EC 2, section 5.2) for isolated columns in braced systems.

$$e_a = \frac{l_0}{400} \rightarrow M_a = N_{Ed} \cdot e_a = N_{Ed} \cdot \frac{l_0}{400}$$

Finally, the nominal second order moment M_2 can be estimated on the basis of the following expression of e_2

$$M_2 = N_{Ed} \cdot e_2 \qquad e_2 = \frac{l_0^2}{c} \cdot \left(\frac{1}{r}\right)$$

l_0 = effective length

$1/r$ = curvature

(r = radius of curvature)

c = factors depending on the curvature distribution

Constant cross sections $\rightarrow c = \pi^2 \cong 10$

π^2 corresponds to a sinusoidal curvature distribution. If it is constant (i.e. the total moment is constant),

$c = 8$

NOTE that c is related to the total curvature (first-order plus second-order)

- To calculate e_2, we need an estimation of $\left(\frac{1}{r}\right)$

Nominal curvature $\frac{1}{r} = K_r \cdot K_\varphi \cdot 1/r_0$

$$\frac{1}{r_0} = \frac{\varepsilon_{yd}}{0.45d}$$

ε_{yd} = yielding design strain of steel $= f_{yd}/E_s$

d = effective depth of the cross section

K_φ [it was called k_1 in class] $= 1 + \beta \varphi_{ef} \geq 1$

φ_{ef} = effective creep ratio $= \underbrace{\varphi(\infty, t_0)}_{\text{final creep coefficient}} \cdot M_{0Eqp}/M_{0Ed}$

M_{0Eqp}: first order moment with quasi-permanent load combination

M_{0Ed} → first order moment for ULS combination

$$\beta = 0.35 + \frac{f_{ck}}{200} - \frac{\lambda}{150}$$

f_{ck}: characteristic compressive strength of concrete

λ → slenderness of the column

$$K_r = \frac{n_u - n}{n_u - n_{bal}} \leq 1$$

$$n = \frac{N_{Ed}}{A_c f_{cd}}$$ relative axial force

A_c → area of concrete, could be taken as $b \cdot h$

$$n_u = 1 + \omega = \frac{A_c f_{cd} + A_s f_{yd}}{A_c f_{cd}} \rightarrow 1 + \underbrace{\omega}_{\frac{A_s f_{yd}}{A_c f_{cd}}}$$

A_s → total area of longitudinal reinforcement
A_c → area of concrete cross section (could be taken as bh)

n_{bal} = value of n at maximum moment resistance

$n_{bal} = 0.4$ may be used

The problem is that, when designing a cross section, we do not know the amount of steel, so $\omega = ?$
Therefore, you need to carry out a trial-and-error procedure, iterating results.
Usually, interaction diagrams N-M report also lines related to various values of K_r.
The first iteration can be performed using $K_r = 1$.
Thus, you can calculate $e_2 \rightarrow M_{Ed} = N_{Ed}(e_0 + e_2 + e_2)$
Then, you get $n_{Ed} = N_{Ed}/A_c f_{ck}$; $m_{Ed} = M_{Ed}/bh^2 f_{ck}$

→ or f_{cd}, depends upon the chart you use

Then, you can place the point n_{Ed}, m_{Ed} inside the chart, and read the value of K_r on the line closer to the point (or interpolating a value of K_r).
You can go back to the calculation of ~~Med~~ e_2 using this second value of K_r, so you can update the value of m_{Ed} using the second estimation of e_2, and place your point n_{Ed}, m_{Ed} in the chart, and read a third value of K_r; if this value is sensibly different from the second, repeat the iteration.
Otherwise, you can stop it.

THE MOMENT MAGNIFICATION METHOD for slender columns

This method is similar to the previous. The aim is to increase the value of bending moment to be used, in order to design a slender column in the same way of a short column, but with enhanced internal forces.

The total design moment M_{Ed} is calculated by multiplying the first order design moment M_{0Ed} (including imperfections, so considering $N_{Ed} \cdot e_2 = N_{Ed} \cdot \frac{l_0}{400}$ too) for the "moment magnification" factor.

$$M_{Ed} = M_{0Ed}\left[1 + \frac{\beta}{\frac{N_B}{N_{Ed}} - 1}\right]$$

M_{0Ed} = first order moment, imperfections included

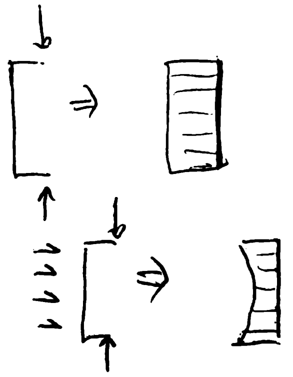
N_{Ed} = design axial load

N_B = buckling load based on nominal stiffness

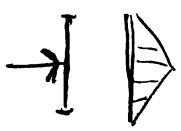
β = factors which depends upon the distribution of first and second order effects.

Calculation of β:
- isolated members with constant cross sections and constant axial load → sine-shape for second-order moment

$\beta = \frac{\pi^2}{c_0}$ $\quad c_0$ → depends upon the distribution of the first order moment



$$c_0 = \begin{cases} 8 \rightarrow \text{constant first order moment} \\ 9.6 \rightarrow \text{parabolic first order moment} \\ 12 \rightarrow \text{symmetric triangular distribution} \end{cases}$$



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- members without transverse load: M_{01} and M_{02} may be replaced by an equivalent (and constant!) moment $M_{0eq} = 0.6\, M_{02} + 0.4\, M_{01} \geq 0.4\, M_{02}$
  Thus, $c_0 = 8$ (constant distribution of moments)
- Members in double curvature $\rightarrow c_0 = 8$
- other cases, different from the previous: $\beta = 1$ is a reasonable simplification.

$$M_{Ed} = M_{0Ed}\left[1 + \frac{\beta \rightarrow 1}{\frac{N_B}{N_{Ed}} - 1}\right] = M_{0Ed}\left(1 + \frac{1}{\frac{N_B - N_{Ed}}{N_{Ed}}}\right)$$

$$= M_{0Ed}\left(1 + \frac{N_{Ed}}{N_B - N_{Ed}}\right) = M_{0Ed}\left(\frac{N_B \cancel{- N_{Ed}} \cancel{+ N_{Ed}}}{N_B - N_{Ed}}\right)$$

$$= \frac{M_{0Ed}}{1 - N_{Ed}/N_B}$$

—— o ——

We need now an estimation of N_B = buckling load based on nominal stiffness

We have to go back to the Euler's buckling load

$$N_E = \frac{\pi^2\, E \cdot I}{l_0^2}\ ;$$

N_B is calculated from N_E, but using a "nominal stiffness" for EI

Nominal stiffness $EI = K_c \cdot E_{cd} \cdot I + K_s\, E_s I_s$

I_c = second moment of area (moment of inertia) of concrete only

I_s = second moment of area of the steel reinforcement

E_{cd} = design value of elastic modulus of concrete

$\hookrightarrow E_{cd} = E_{cm}/\gamma_{CE} \longrightarrow \gamma_{CE}$ can be taken as 1.2

K_c is a factor that accounts for effects of cracking, creep, etc.

K_s is a factor that accounts for the contribution of the reinforcem.

E_s is the Young's modulus of steel.

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(1) • If $\rho = \dfrac{A_s}{A_c}$ $A_s \rightarrow$ total area of steel; $A_c \rightarrow$ area of concrete

$\rho \geq 0.002$ (~~[illegible]~~)

$K_s = 1$; $K_c = k_1 \cdot k_2 / (1 + \varphi_{ef})$ $\varphi_{ef} \rightarrow$ effective creep ratio $\varphi_{ef} = \varphi(\infty, t_0) \dfrac{M_{0Eqp}}{M_{0Ed}}$

$k_1 = \sqrt{\dfrac{f_{ck}}{20}}$ $f_{ck} \rightarrow$ characteristic compressive strength, in N/mm^2

$k_2 = n \cdot \dfrac{\lambda}{170} \leq 0.20$ $\leftarrow n = \dfrac{N_{Ed}}{A_c f_{cd}}$

λ = slenderness ratio of the column

if λ is not defined, $k_2 = n \cdot 0.30$
$\downarrow$ must be ≤ 0.20

(2) • If $\rho \geq 0.01$ $\left(\rho = \dfrac{A_s}{A_c} \right)$

$K_s = 0$; $K_c = \dfrac{0.3}{1 + 0.5\,\varphi_{ef}}$

$\rightarrow$ condition to apply this simplified formulation, however the previous remains still valid!

SIMPLIFIED ALTERNATIVE

(1) remains valid!
(2) can be used in a preliminary step, before a more refined calculation according to (1)

NOTE In statically indeterminate structures, the unfavourable effects of cracking of adjacent members should be taken into account.
As a simplification, fully cracked sections may be assumed.
The stiffness should be based on a reduced value of elastic modulus $E_{cd,eff}$

$$E_{cd,eff} = \frac{E_{cd}}{1 + \varphi_{ef}} = \frac{E_{cm} / \gamma_{cE}}{1 + \varphi_{ef}}$$

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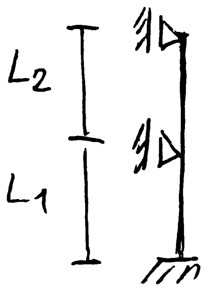
Page 13 EXAMPLE – short or slender?



$L_1 = 4\,m$

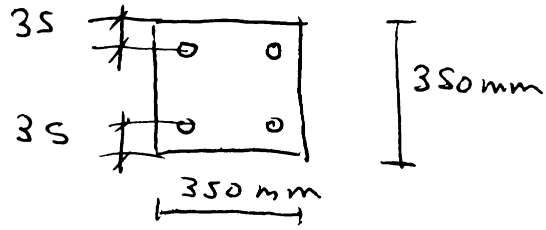
$L_2 = 3.2\,m$

$N_{Ed} = 223\,kN$

$M_{Ed} = 180\,kN \cdot m$

concrete: C30/37

steel: $f_{yk} = 430\,MPa$



Step 1

Design strengths

$$f_{cd} = \frac{0.85\, f_{ck}}{\underbrace{1.50}_{\gamma_c}} = \frac{0.85 \cdot 30}{1.50} = 17\ N/mm^2$$

$$f_{yd} = \frac{f_{yk}}{\underbrace{1.15}_{\gamma_s}} = 374\ N/mm^2$$

Step 2

Slenderness ratio

$L_1 \neq L_2 \Rightarrow$ different lengths, different boundary conditions $\Big\}\ l_{0,1} \neq l_{0,2}$

$i = \sqrt{I/A} \rightarrow$ square column, same i in both the principal directions

$$I = \frac{1}{12} b h^3 \Rightarrow \frac{b^4}{12}\ ;\ A = b^2 \rightarrow i = \frac{b}{\sqrt{12}} = 101\,mm$$

$$l_{0,1} = 0.7 \cdot L_1 = 2.8\,m \rightarrow \lambda_1 = \frac{2.8 \cdot 10^3}{101} = 27.7$$

$$l_{0,2} = 1 \cdot L_2 = 3.2\,m \rightarrow \boxed{\lambda_2 = 31.7} \leftarrow \text{max } \lambda\,!$$

Step 3 limit value λ_{lim}

$$\lambda_{lim} = 20 \cdot A \cdot B \cdot C / \sqrt{n}$$

$$n = N_{Ed}/A_c f_{cd} = \frac{223 \cdot 10^3}{350^2 \cdot 17} = 0.107$$

$A = \dfrac{1}{1 + 0.2\,\varphi_{ef}}$ $\quad \varphi_{ef}$ not known $\rightarrow A = 0.7$

$B = \sqrt{1 + 2\omega}$ $\quad \omega$ not known $\rightarrow B = 1.1$

$C = 1.7 - r_m$ $\quad r_m = \dfrac{M_{01}}{M_{02}}$

r_m unknown, take the most conservative value $\rightarrow C = 0.7$

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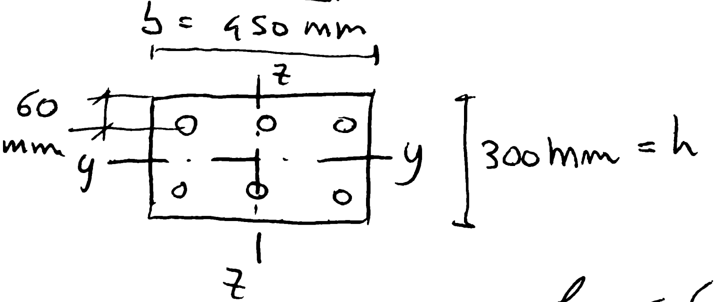
therefore: $\lambda_{lim} = 20 \cdot \underset{A}{0.7} \cdot \underset{B}{1.1} \cdot \underset{C}{0.7} / \sqrt{\underset{n}{0.107}} = 32.9$

$\max \lambda = \lambda_2 = 31.7 < 32.9 = \lambda_{lim}$

Thus, the column is short → use interaction diagrams with N_{Ed}, $M_{Ed} = M_{0Ed} + M_a$

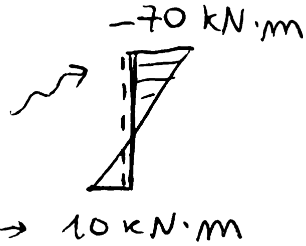
M_a → imperfections

EXAMPLE



single bending moment M_y

$N_{Ed} = 1700$ kN



Concrete C25/30

Steel $f_{yk} = 500$ MPa

$l_{0,y} = 6.75$ m

$l_{0,z} = 8$ m

$\varphi_{ef} = 0.87$

Step 1 Design strengths: $f_{cd} = \dfrac{0.85 f_{ck}}{1.50} = 14.2 \ N/mm^2$

$f_{yd} = f_{yk}/1.15 = 435 \ N/mm^2$

Step 2 Slenderness ratio (check both planes y-y and z-z!)

$i_y = \sqrt{I/A} = \sqrt{\frac{1}{12} bh^3 / bh} = \dfrac{h}{\sqrt{12}} = 86.6$ mm

$i_z = \sqrt{\frac{1}{12} b^3h / bh} = \dfrac{b}{\sqrt{12}} = 129.9$ mm

$\lambda_y = \dfrac{l_{0,y}}{i_y} = \dfrac{6.75 \cdot 10^3}{86.6} = 77.9$ ← $\boxed{\lambda_{max} = \lambda_y!}$

$\lambda_z = l_{0,z}/i_z = 8 \cdot 10^3 / 129.9 = 61.6$

the plane y-y is more likely to buckle

Step 3 limit slenderness value

$\lambda_{lim} = 20 \times A \times B \times C / \sqrt{n}$

$n = \dfrac{N_{Ed}}{A_c \cdot f_{cd}} = \dfrac{1700 \cdot 10^3}{300 \cdot 450 \cdot 14.2} = 0.887$

$$A = \frac{1}{1+0.2\,\varphi_{ef}} = \frac{1}{1+0.2\cdot 0.87} = 0.852$$

$$B = \sqrt{1+2\omega} \qquad \omega \text{ not known} \rightarrow B = 1.1$$

$$C = 1.7 - r_m = 1.7 - \left(\frac{M_{01}}{M_{02}}\right) = 1.7 - \left(-\frac{10}{70}\right)$$

0.143

$$= 1.843$$

$$\lambda_{lim} = \underbrace{20 \times 0.852 \times 1.1 \times 1.843}_{34.55} \Big/ \sqrt{0.887} = \boxed{36.7}$$

$$\lambda_{max} = \lambda_y = 77.9 > \lambda_{lim} = 36.7 \Rightarrow \underline{\text{SLENDER}}$$

<u>step 4</u> different end moments $\rightarrow$ equivalent eccentricity

$$e_{0Eq} = 0.6\, e_{02} + 0.4\, e_{01} \geq 0.4\, e_{02}$$

$$= 0.6 \cdot \frac{M_{02}}{N_{Ed}} + 0.4\left(-\frac{M_{01}}{N_{Ed}}\right) = \left[\begin{array}{l}\text{note that } M_{01} \text{ and } M_{02} \\ \text{have opposite sign!}\end{array}\right]$$

$$= 0.6\, \frac{70\ \mathrm{kN\cdot m}}{1700\ \mathrm{kN\cdot m}} - 0.4\, \frac{10\ \mathrm{kN.m}}{1700\ \mathrm{kN\cdot m}} = 0.0247 - 0.0024$$

$$= 22.3\ \mathrm{mm}$$

$$e_{0Eq} \geq 0.4\, e_{02}\ ? \qquad 0.4\, e_{02} = 16.5\,\mathrm{mm} < 22.3 \quad \underline{\text{OK}}$$

eccentricity due to imperfections

$$e_a = \frac{l_0}{400} = \frac{l_{0,y}}{400} = 16.9\ \mathrm{mm}$$

$$\varepsilon_{yd} = f_{yd}/E_s = \frac{f_{yk}}{\gamma_s E_s}$$

Second order eccentricity

$$e_2 = \frac{K_\varphi \cdot K_r \cdot l_0^2}{\pi^2}\left(\frac{\varepsilon_{yd}}{0.45d}\right) = \frac{K_\varphi K_r\, l_0^2}{\pi^2}\left(\frac{f_{yk}}{\gamma_s \cdot E_s \cdot 0.45d}\right)$$

$$= \frac{K_\varphi \cdot K_r \cdot l_{0,y}^2}{\pi^2} \cdot \frac{f_{yk}}{1.15\cdot 200\cdot 10^3 \cdot 0.45\cdot d} \cong \frac{K_\varphi K_r \cdot l_{0y}^2 \cdot f_{yk}}{\pi^2 \cdot 103500\, d}$$

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$$K_\varphi = 1 + \beta\,\varphi_{ef} \geq 1 \qquad \beta = 0.35 + \frac{f_{ck}}{200} - \frac{\lambda}{150}$$

$$\beta = 0.35 + \frac{25}{200} - \frac{\overset{\lambda_y}{77.9}}{150} = -0.044$$

$$k_\varphi = 1 - 0.044 \cdot 0.87 \leq 1 \Rightarrow K_\varphi = 1$$

$$K_r = \frac{n_{ud} - n_{Ed}}{n_{ud} - n_{bal}} \qquad n_{Ed} = N_{Ed}/A_c f_{cd} = \frac{1700 \cdot 10^3}{450 \times 300 \times 14.2} = 0.887$$

you cannot calculate K_r, since ω is not yet known!

$$n_{ud} = 1 + \omega$$

$$n_{bal} = 0.4$$

* 1st trial: $K_r = 1 \longrightarrow e_2 = \dfrac{1 \cdot 1 \cdot \overset{l_{0,y}}{6750}^2 \cdot \overset{f_{yk}}{500}}{\pi^2 \cdot 10^3 \cdot 500 \cdot \underbrace{(300-60)}_{d}} = 92.9\ \text{mm}$

$$M_{Ed} = M_{0eq} + M_a + M_2 = N_{Ed}\,(e_{0,eq} + e_a + e_2)$$

$$= 1700 \cdot 10^3 \,(\underset{e_{0,eq}}{22.3} + \underset{e_a}{16.9} + 92.9)\ \text{N}\cdot\text{mm}$$

$$= 1700 \cdot 10^3 \cdot 132.1 = 224.57\ \text{kN}\cdot\text{m}$$

$$m_{Ed} = \frac{224.57 \cdot 10^6}{300^2\, 450\, (25)} = 0.222$$

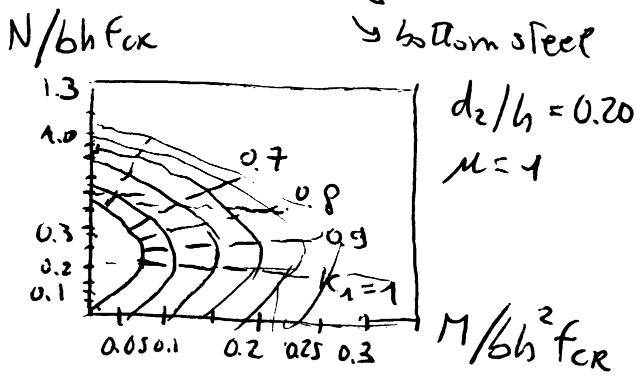
25 → f_{ck} for diagrams (depending upon the book...)

$$n_{Ed} = \frac{1700 \cdot 10^3}{300 \times 450 \times (25)} = 0.504$$

Let's consider a symmetric reinforcement $\Rightarrow \mu = \dfrac{A_s'}{A_s} = 1$ (A_s' → top steel, A_s → bottom steel)

calculate $d'/h = 60/300 = 0.2$

Place the point $m_{Ed} = 0.222$ and $n_{Ed} = 0.504$ in the proper interaction diagram



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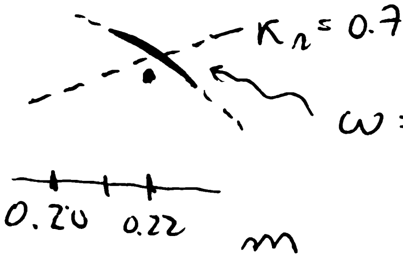
$n \quad 0.5$



$\omega = \dfrac{A_s \cdot f_{yk}}{bh f_{ck}} = 0.9$

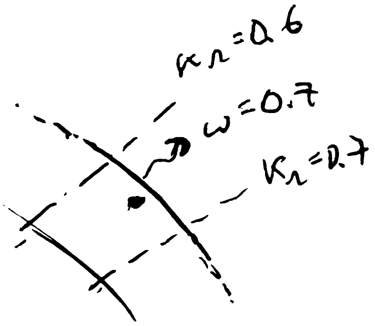
You will get a point (calculated using $K_r = 1$) close to the line $K_r = 0.7$

2nd trial use $K_r = 0.7$ (or an interpolated value between two lines) to calculate again e_2

$$e_2 = \frac{1 \cdot 0.7 \cdot 6750^2 \cdot 500}{\pi^2 \cdot 103500 \cdot 240} = 65.0 \text{ mm}$$

$$M_{Ed} = N_{Ed}(e_{0eq} + e_a + e_2) = 1700 \cdot 10^3 \underbrace{(22.3 + 16.9 + 65.0)}^{104.2} = 177.14 \text{ kN}\cdot\text{m}$$

$$m_{Ed} = \frac{177.14 \cdot 10^6}{450 \cdot 300^2 \cdot 25} = 0.175$$



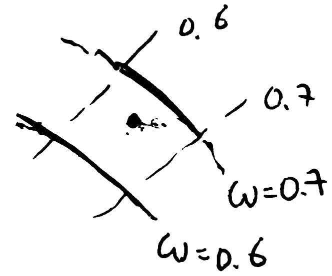
$n_{Ed} \rightarrow$ it remains the same

place the point $n_{Ed} = 0.504$, $m_{Ed} = 0.175$
It stands behind the line $\omega = 0.7$, and in the middle between $K_r = 0.6$ and $K_r = 0.7$

3rd trial use $K_r = 0.65$ $\rightarrow e_2 = \dfrac{1 \cdot 0.65 \cdot 6750^2 \cdot 500}{\pi^2 \cdot 103500 \cdot 240} = 60.4$ mm

update $M_{Ed} = 1700 \cdot 10^3 \underbrace{(22.3 + 16.9 + 60.4)}_{99.6} = 169.32$ kN·m

update $m_{Ed} = \dfrac{M_{Ed}}{bh^2 f_{ck}} = \dfrac{169.32 \cdot 10^6}{450 \cdot 300^2 \cdot 25} = 0.167$



Again, the point stands in the middle between $K_r = 0.7$ and $K_r = 0.6$ $\rightarrow$ no need for more iterations, since the value we used is very close to the current value

$\rightarrow$ minimum area of steel: $\omega = 0.7$ (the first interaction diagram which includes our design point.

$\omega = 0.7 \rightarrow \dfrac{A_s \cdot f_{yk}}{bh f_{ck}} = 0.7 \rightarrow A_{s,min} = 0.7 \cdot bh f_{ck} / f_{yk}$

it depends upon the diagram that you use!

total, for those diagrams

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$A_{s,min} = 0.7 \cdot 450 \cdot 300 \cdot 25\cancel{00} / \cancel{2500} = 4725 \text{ mm}^2$

$= 4725 \text{ mm}^2 \rightarrow$ total area

$A'_s = \frac{1}{2} A_{s,min} = A_s = 2363 \text{ mm}^2$

$1\phi16 \rightarrow 201 \text{ mm}^2 \rightarrow 11.76 \rightarrow 12$

$1\phi20 \rightarrow 314 \text{ mm}^2 \rightarrow 7.5 \rightarrow 8$

$1\phi24 \rightarrow 452 \text{ mm}^2 \rightarrow 5.2 \rightarrow 6$

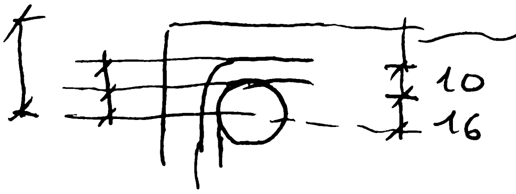
$1\phi25 \rightarrow 491 \text{ mm}^2 \rightarrow 4.8 \rightarrow 5$

$1\phi32 \rightarrow 804 \text{ mm}^2 \rightarrow 2.9 \rightarrow 3 \leftarrow$ Let's use $3\phi32$ per side

$A_{s,tot} = 804 \times 6 = 4824 \text{ mm}^2 > A_{s,min} = 4725 \text{ mm}^2$

Check the cover

Assuming to use stirrups $\phi10$



$\rightarrow 60 - (10 + 16) = 34 \text{ mm}$

The clear cover is 34 mm, which seems OK (of course, you should check it!)

So: should you change the cross section sizes or move the rebars to comply with the minimum cover? Probably not $\rightarrow$ d', d do not vary $\rightarrow$ the geometry of the cross section remains the same $\rightarrow$ no need to double-check the actual section which should be implicitely verified.

However, you can repeat the calculation of e_2, using the actual value of ω

ω (for the calculation of K_r) $= \dfrac{A_{s,tot} \cdot f_{yd}}{A_c \, f_{cd}} = \dfrac{4824 \cdot 435}{300 \cdot 450 \cdot 14.2} = 1.095$

Check if there is too much steel: $A_{s,max} = 0.04 A_c = 0.04 \cdot 300 \cdot 450 = 5400 \text{ mm}^2$ OK $> A_s$

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$$k_r = \frac{n_u - n_{Ed}}{n_u - n_{bal}}$$

$$n_u = 1 + \omega = 1 + 1.095 = 2.095$$

$$n_{Ed} = \frac{1700 \cdot 10^3}{450 \cdot 300 \cdot 14.2} = 0.887$$

$$n_{bal} = 0.4$$

$$\frac{n_u - n_{Ed}}{n_u - n_{bal}} = \frac{2.095 - 0.887}{2.095 - 0.4} = \frac{1.208}{1.695} = 0.713 = k_r \leq 1$$

$$e_2 = \frac{1 \cdot 0.713 \cdot 6750^2 \cdot 500}{\pi^2 \cdot 107500 \cdot 240} = 66.3 \text{ mm}$$

$$M_{Ed} = 1700 \cdot 10^3 \underbrace{(22.3 + 16.9 + 66.3)}_{105.5} = 179.35 \text{ kN} \cdot \text{m}$$

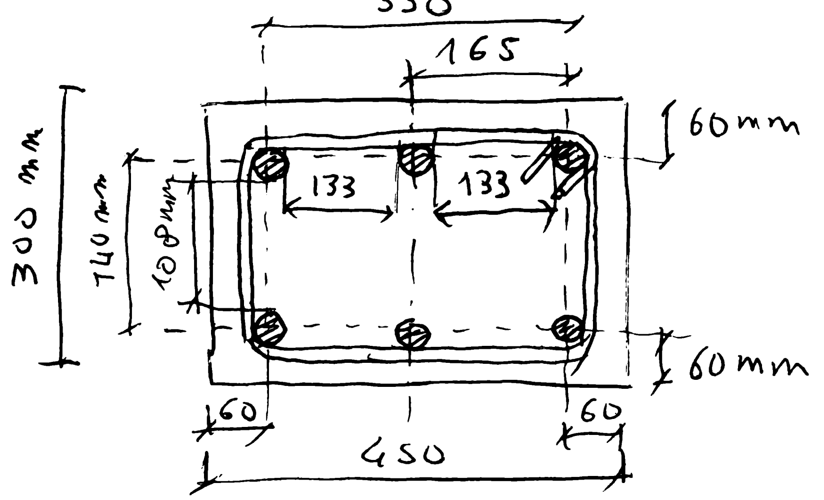
$$m_{Ed} = \frac{M_{Ed}}{b \cdot h^2 f_{ck}} = \frac{179.35 \cdot 10^6}{450 \cdot 300^2 \cdot 25} = 0.177$$

$m_{Ed} < 0.504 \longrightarrow$ for the diagram, f_{ck} has been used! if you use f_{cd}, you get 0.887!

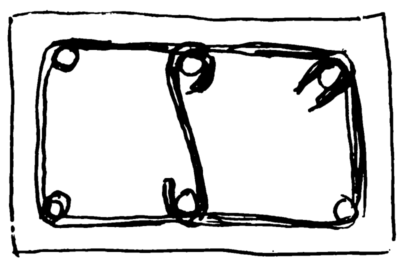
The point stands behind $\omega = 0.7 \rightarrow A_{s,actual} > A_{s,min}$

Thus the section is implicitly verified

Finally: check the arrangement of rebars



The central bars should be restrained by a link



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