MASONRY STRENGTENING WITH COMPOSITES — Padova, 28/06/2012

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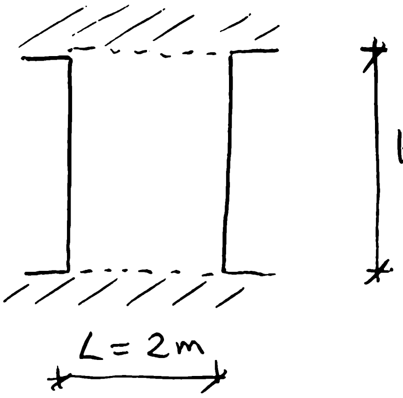
Notes by MATTEO PANIZZA

EXAMPLE: Strengthening of a masonry pier

(taken from Focacci 2007)

Code reference CNR DT 200/2004



WALL GEOMETRY

height $H = 4\ m$

length $L = 2\ m$

thickness $t = 0.3\ m$

MASONRY PROPERTIES

Characteristic compressive strength $f_{mk} = 4\ MPa$

Secant elastic modulus $E_m = 4000\ MPa$

Initial shear strength $f_{vk0} = 0.15\ MPa$

Friction coefficient $\mu = 0.4$

Specific weight $\varrho = 18\ kN/m^3$

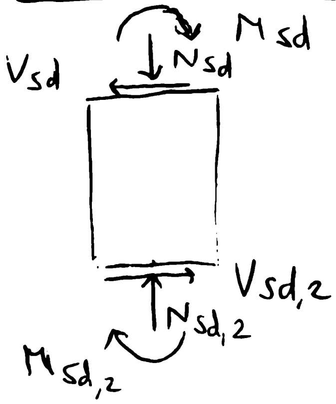
Partial factor for masonry $\gamma_M = 2$

Design properties

$$f_{md} = f_{mk}/\gamma_M = 2\ MPa$$

$$f_{vd} = f_{vk}/\gamma_M = \frac{f_{vk0}}{\gamma_M} + \frac{\mu}{\gamma_M}\sigma_m = 0.075 + 0.2\,\sigma_m \quad [MPa]$$

EXTERNAL FORCES ON THE TOP SECTION (given)



$N_{sd} = 200\ kN$; $V_{sd} = 100\ kN$; $M_{sd} = 250\ kN$

bottom section (calculated)

$N_{sd,2} = N_{sd} + W$ → self weight of the wall, given by density × volume

$= 243.2\ kN$ $\quad W = \varrho \cdot H \cdot L \cdot t = 43.2\ kN$

$V_{sd,2} = V_{sd} = 100\ kN$ to keep the equilibrium

$$M_{sd,2} = V_{sd} \cdot H - M_{sd} = 100 \cdot 4 - 250\ kN \cdot m = 150\ kN \cdot m$$

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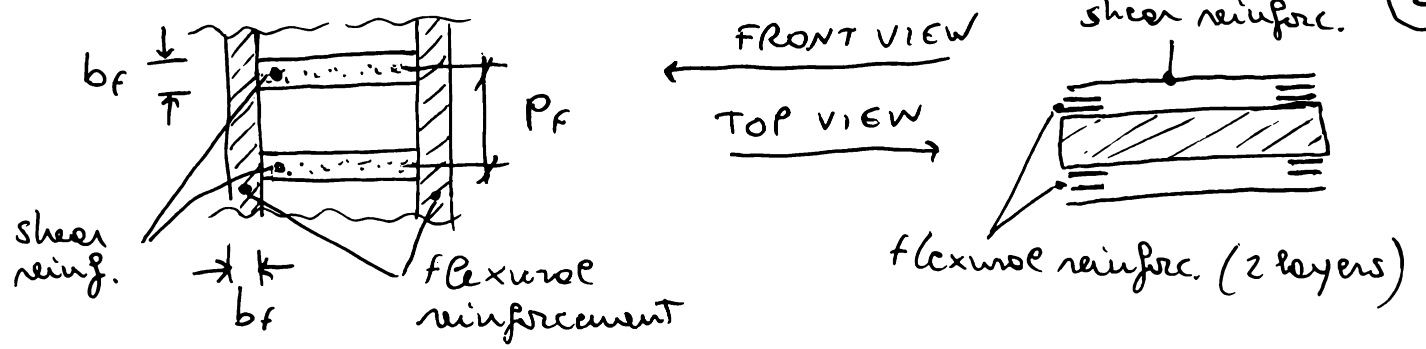
REINFORCEMENT CONFIGURATION



suppose to adopt: - carbon FRP (CFRP)

- a symmetrical configuration for both vertical and horizontal reinforcement
- two layers for the vertical reinforc. and one layer for the horizontal one
- all strips 250 mm wide
- horizontal strips 1 m spaced

CFRP elastic modulus $E_f = 240$ GPa
CFRP charact. ultimate strain $\varepsilon_{fk} = 1.5\%$
" equivalent thickness $t_f = 0.17$ mm

reinforcement width $b_f = 250$ mm
spacing of horizontal reinf. $p_f = 1$ m

DESIGN LIMITS FOR THE CFRP REINFORCEMENT

strain limit = min { max tensile strain; ~~max~~ debonding strain }

<u>tensile limit</u>: max strain = $\underbrace{\eta_a}_{\text{conversion factor (environmental conditions)}} \cdot \dfrac{\varepsilon_{fk}}{\gamma_f}$

ε_{fk} → characteristic ultimate strain (given by datasheet)

γ_f → partial factor for tensile failure

CNR-DT200/2004

§ 3.4.1, Table 3-2 (application type A): $\gamma_f = 1.1$

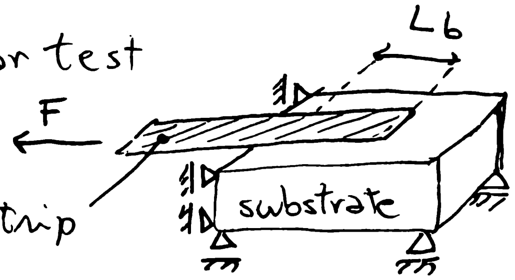
§ 3.5.1, Table 3-4 (external environment, CFRP): $\eta_a = 0.85$

$$\eta_a \cdot \frac{\varepsilon_{fk}}{\gamma_f} = 0.85 \cdot \frac{1.5 \cdot 10^{-2}}{1.1} = 1.16\ \%$$

debonding limit

In most cases, epoxy composites detach from the substrate before reaching their tensile strength.
The detachment can occur at the end of the bonded area (plate-end debonding) or at an internal position (intermediate debonding, e.g. at a shear or flexural crack, if you consider a strengthened R.C. beam)

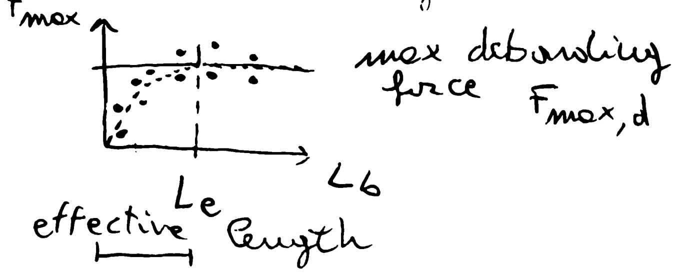
shear test



FRP strip

L_b = bonded length

F_{max} = max tensile force before debonding



If $L_b \geq L_e$, F_{max} doesn't increase

Provided that $L_b \geq L_e$: $F_{max,d} = \underbrace{b_f}_{\text{reinforcement's width}} \cdot \sqrt{2\,\underbrace{\Gamma_f}_{\text{interface fracture energy}} \cdot \underbrace{E_f \cdot t_f}_{\text{reinforcement axial stiffness per unit width}}}$

This formula in commonly found in literature, and it's valid when substrate axial stiffness is very large compared to FRP stiffness.

Note that models were developed for concrete substrates, then applied, with little modifications, to masonry supports.
At present, the role of mortar joints is not yet taken into account, due to a certain lack of experimentations.
Therefore masonry is treated as a homogeneous material

§ 5.3.2, Eq (5.3) $\underbrace{\Gamma_{Fk}}_{\text{characteristic value of fracture energy}} = c_1 \cdot \sqrt{\underbrace{f_{mk}}_{\text{characteristic compressive strength of masonry}} \cdot \underbrace{f_{mtm}}_{\text{masonry average tensile strength}}}$

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f_{mtm} DT 200/2004, Italian version, says that f_{mtm}, actually, should be the masonry <u>unit</u> tensile strength (that is, brick or stone strength).
2012 Revision 1 version corrects the ambiguity (anyway missing from the 2004 English version due to a probable mistyping) by telling that both f_{mk} and f_{mtm} refer to the units and not the masonry ensemblage.

However, in absence of specific data (experimental tests, ...)

$f_{mtm} = 0.1\, f_{mk}$

$c_1 = 0.015 \longrightarrow$ note that it's half the value proposed for k_G (0.03) in the case of concrete substrate (§4.1.2, Eq(4.2))

The document was released in 2004, based on earlier State-of-Art (very few experimentations related to masonry substrates)

<u>concrete</u> $\Gamma_{Fk} = k_b \cdot k_G \sqrt{f_{ck} \cdot f_{ctm}}$

$k_G \rightarrow 0.03$

$k_b \rightarrow \sqrt{\dfrac{2 - b_f/b}{1 + b_f/400}}$

b_f = FRP width
b = RC member width

400 should be 400 mm, otherwise the k_b coefficient wouldn't be adimensional

400 probably refers to a common RC beam width
[see fib Bulletin 14]

<u>masonry</u> c_1 instead of $k_b \cdot k_G$ because it was difficult, at that time, to express a formulation for the "scale" coefficient k_b, in the case of a masonry wall

2012 Rev 1: the expression is going to change, adopting two coefficients also for masonry substrates

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2012 R1 $\underbrace{\Gamma_{Fd}}_{\text{design value!}} = \dfrac{k_b \cdot k_G}{FC} \sqrt{f_{bm} \cdot f_{btm}}$

f_{bm}, f_{btm} = average compressive and tensile strength of masonry units, respectively

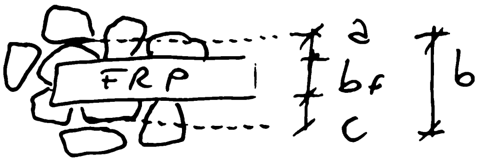
FC = confidence factor (related to the level of knowledge of structure's properties)

$$k_b = \sqrt{\frac{3 - b_f/b}{1 + b_f/b}}$$

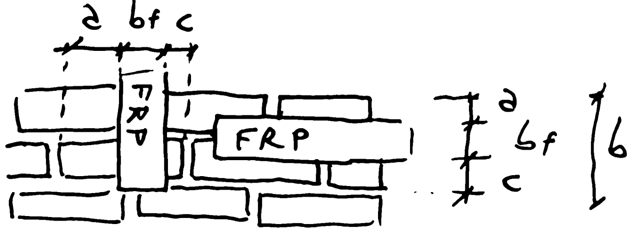
b_f = FRP width

$b = b_f + b_d$ influenced area

$b_d = a + c \rightarrow \begin{cases} \text{average stone diameter} \\ \text{orthogonal brick dimension} \end{cases}$



stone masonry



brick masonry

$$\underbrace{k_G}_{} = \begin{cases} 0.031 \text{ mm} & \text{clay masonry} \\ 0.048 \text{ mm} & \text{tuff masonry} \\ 0.012 \text{ mm} & \text{calcarenite / leccese stone masonry} \end{cases}$$

↳ values provided for Externally Bonded Fabrics (on-site impregnation)

Pultruded laminates: 40% reduction of k_G

$$k_{G,P.L.} = 0.6\, k_G \longrightarrow \begin{cases} 0.0186 & \text{clay} \\ 0.0288 & \text{tuff} \\ 0.0072 & \text{calcarenite} \end{cases}$$

Summing up:

$$\Gamma_{Fk} = c_1 \cdot \sqrt{f_{mk} \cdot f_{mtm}} = 0.015 \cdot \sqrt{f_{mk} \cdot 0.1\, f_{mk}}$$

$$\overset{!}{=} 0.015 \cdot 0.316 \cdot 4 = 18.96 \cdot 10^{-3}\ N/mm$$

↳ units are not consistent, since c_1 should be expressed in mm

$$\Gamma_{Fd} = \frac{\Gamma_{Fk}}{\gamma_m} = \frac{18.96 \cdot 10^{-3}}{2} N/mm = 9.48\ N/mm$$

↳ design value. Fracture energy is a material property, so γ_m is the partial factor for masonry

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max design force for the FRP, considering debonding failure :

$$P_{fdd} = \underbrace{\frac{1}{\gamma_{f,d}}}_{\text{partial factor for debonding mechanism}} \cdot \sqrt{2 \cdot E_f \cdot t_f \cdot \Gamma_{Fd}}$$

force per unit width (b_F is missing)

t_f → equivalent thickness of dry fabric

E_f → tensile elastic modulus, as given by producer's datasheet

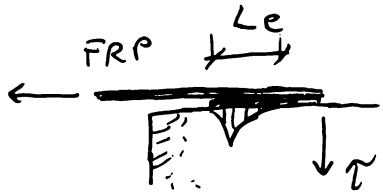
Partial factor for debonding mechanism

$\gamma_{f,d} = 1.2$ (§3.4.1, table 3-2)

$E_f \cdot t_f$ = FRP stiffness per unit width

the higher the stiffness, the greater the debonding force

stiffer composites : larger effective lengths L_e, and better performances in terms of force



low stiffness → short L_e → more stress concentration

high stiffness → long L_e → more stress distribution

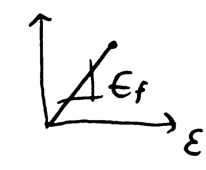
$$P_{fdd} = \begin{cases} \text{vertical reinforcement: } t_f = 2 \cdot 0.17\ mm = 0.34\ mm \\ \text{horizontal reinforcement: } t_f = 0.17\ mm \end{cases}$$

$$P_{fdd} = \begin{cases} \frac{1}{1.2} \cdot \sqrt{2 \cdot 240 \cdot 10^3 \frac{N}{mm^2} \cdot 0.34\ mm \cdot 9.49 \cdot 10^{-3} \frac{N}{mm}} \\ \frac{1}{1.2} \cdot \sqrt{2 \cdot 240 \cdot 10^3 \frac{N}{mm^2} \cdot 0.17\ mm \cdot 9.49 \cdot 10^{-3}\ N/mm} \end{cases}$$

$$= \begin{cases} 32.8\ N/mm & \text{vertical reinforcement} \\ 23.2\ N/mm & \text{horizontal reinforcement} \end{cases}$$

debonding strain

$$\varepsilon_{fdd} = \frac{\overbrace{P_{fdd} \cdot b_F}^{\text{max force}}}{\underbrace{b_f \cdot t_f}_{\text{fibres area}} \cdot E_f}$$



reinforcement elastic till failure

$$\varepsilon_f = \frac{\sigma_f}{E_f}$$

$$\varepsilon_{fdd} = \begin{cases} \frac{32.8\ N/mm}{0.34 \cdot 240 \cdot 10^3\ N/mm} = 0.402\ ‰ & \text{vertical reinf.} \\ \frac{23.2\ N/mm}{0.17\ mm \cdot 240 \cdot 10^3\ N/mm^2} = 0.569\ ‰ & \text{horizontal r.} \end{cases}$$

limit tensile strain $1.16\,\% = 11.6\,‰$
for both reinforcements

Then, ε_{fdd} is the maximum admissible strain as it happens in most cases.

Combined compression and bending – top section

$N_{sd} = 200\ kN$

$V_{sd} = 100\ kN$

$M_{sd} = 250\ kN \cdot m$

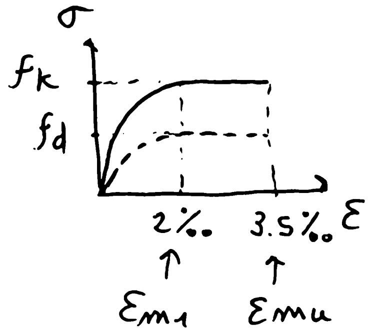
eccentricity of vertical load

$$e = \frac{M_{sd}}{N_{sd}} = \frac{250\ kNm}{200\ kN} = 1.25\,m$$

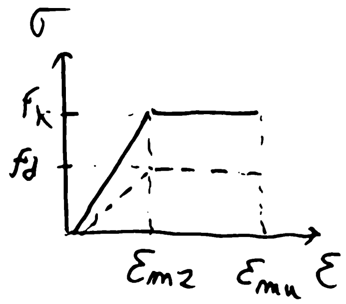
The vertical load falls outside the section, so it cannot be withstood by the unreinforced masonry

Possible stress-strain laws for masonry

parabolic-rectangular (EC6 – § 3.7)

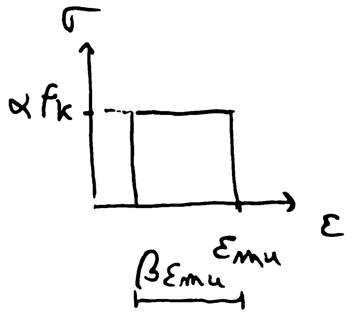


Linear-rectangular



$\varepsilon_{m2} = \dfrac{f_k}{E}$ or $\varepsilon_{m2} = \varepsilon_{m1}$

stress-block

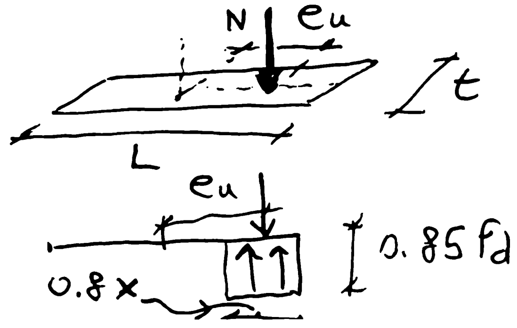


The stress-block law is proposed by the Italian Building Code (§ 7.8.3.2.1) for steel reinforced masonry, adopting $\begin{cases} \alpha = 0.85 \\ \beta = 0.8 \end{cases}$

The same stress-block is also used by the code to calculate the flexural strength of unreinforced masonry, even if it's not explicitly written (Eq. 7.8.2)

$$M_u = \frac{L^2 t\, \sigma_0}{2}\left(1 - \frac{\sigma_0}{0.85\, f_d}\right)$$

ultimate moment for unreinforced masonry



$M_u = N \cdot e_u$ $\qquad \sigma_0 = \dfrac{N}{L \cdot t}$

$$e_u = \frac{L}{2} - \frac{1}{2}(0.8\,x)$$

$\beta = 0.8$

$$N = \underbrace{0.85 f_d}_{\cancel{\alpha} = 0.85} \cdot \underbrace{0.8 x \cdot t}_{\text{area}} \rightarrow x = \frac{N}{0.8 \cdot 0.85 \cdot t \cdot f_d} \qquad f_d \rightarrow f_k/\gamma_M$$

$$e_u = \frac{L}{2} - 0.4 \cdot \frac{N}{0.8 \cdot 0.85 \cdot t \cdot f_d}$$

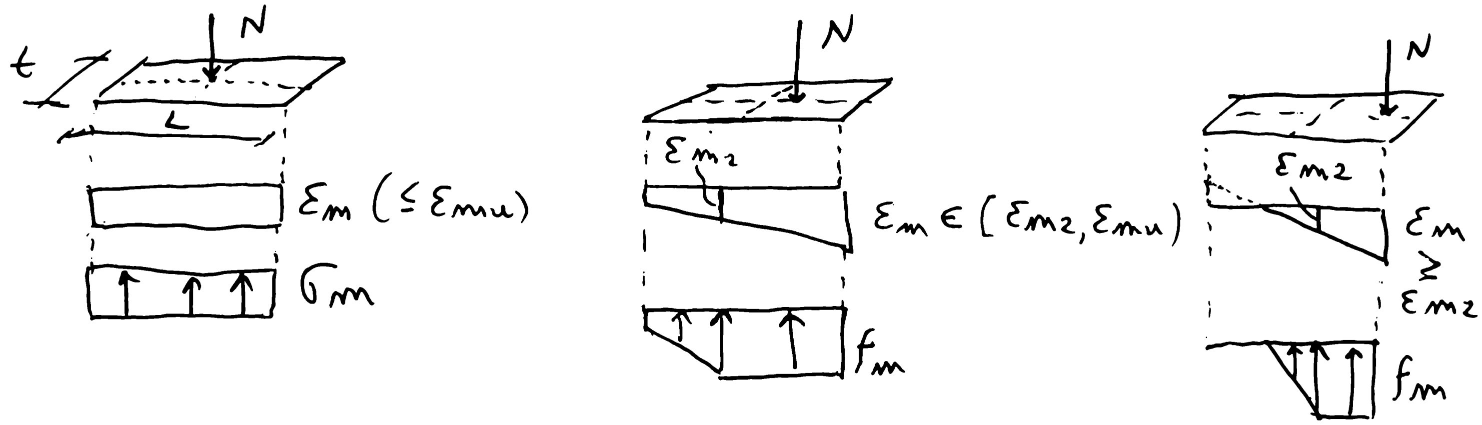
$$M_u = N \cdot e_u = \frac{NL}{2} - \frac{0.4 N^2}{\cancel{0.8}_{2} \cdot 0.85 \cdot t \cdot f_d} = \frac{NL}{2}\left(1 - \frac{N}{L \cdot 0.85 t f_d}\right)$$

$$= \frac{\sigma_0 t \cdot L^2}{2}\left(1 - \frac{\sigma_0 \cancel{tL}}{\cancel{tL} \cdot 0.85 f_d}\right) = \frac{L^2 t \sigma_0}{2}\left(1 - \frac{\sigma_0}{0.85 f_d}\right)$$

Eq. 7.8.2 obtained by using a stress block with $\alpha = 0.85$, $\beta = 0.8$

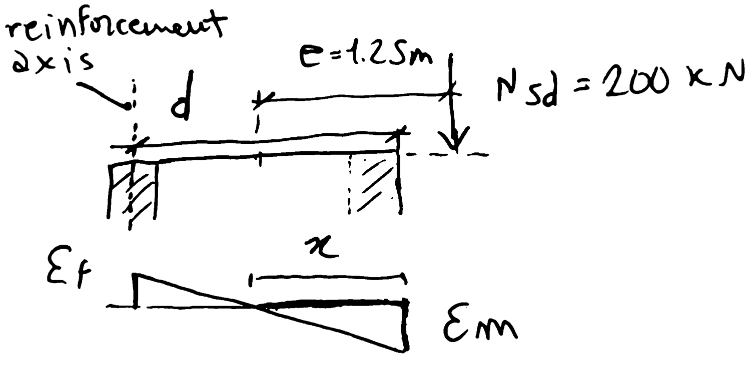
FRP reinforced section: if the eccentricity is lower than half the section, reinforcement is not necessary since the unreinforced masonry can withstand a bending moment

Some possible situations (adopting a linear-rectangular law)



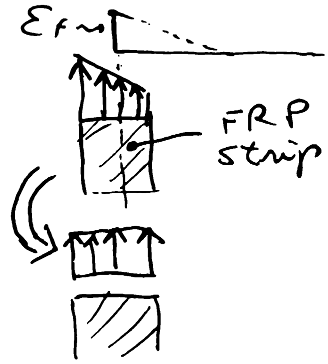
Even if the section is only partially compressed, you can consider that reinforcement could not be necessary

Anyway, on the top section $e = 1.25\ m > \frac{L}{2} = 1\ m$



In practice, the problem is simplified this way:

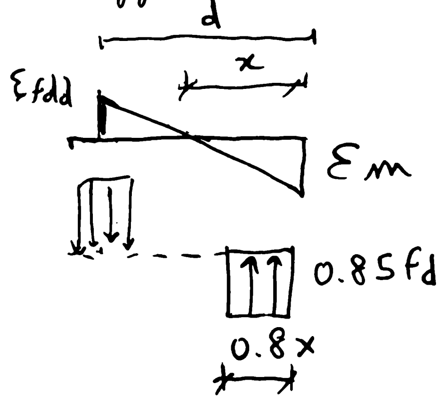
- only FRP in tension are considered
- properties refer to FRP axis, ~~on~~ adopting one strain and one average stress



To evaluate M_{Rd}, the maximum resisting moment, you may adopt a complex law or a stress-block.

<u>Using a stress-block</u> $\alpha = 0.85$; $\beta = 0.8$

Suppose the FRP failure due to $\varepsilon_f = \varepsilon_{fdd} = 0.402\,‰$



<u>Note</u> that using a stress-block means that you consider only equilibrium, not congruence. The compressed area is $\beta \cdot x$ (x = neutral axis depth), no matter what the ε_m is (ε_m = strain at the compressed masonry edge)

$$\underbrace{d}_{} = L - \frac{1}{2} b_f = \left(2 - \frac{0.25}{2}\right) m = 1.875\,m$$

distance between FRP axis and compressed edge

<u>Equilibrium</u>: $C - T = N_{sd}$

C = compression on masonry
T = tension on FRP

$$C = \underbrace{\alpha f_{md}}_{\text{stress on masonry}} \cdot \underbrace{\beta x \cdot t}_{\text{area}} \qquad T = \underbrace{\varepsilon_{fdd} \cdot E_f}_{\text{stress on the FRP}} \cdot \underbrace{A_f}_{\text{area given by } b_{f,td} \cdot t_f}$$

$$0.85 \cdot f_{md} \cdot 0.8 \cdot x \cdot t = \varepsilon_{fdd} \cdot E_f \cdot A_f + N_{sd}$$

$A_f = 2 \cdot 250\,mm \cdot 2 \cdot 0.17\,mm \doteq 170\,mm^2$

$$x = \frac{\varepsilon_{fdd} \cdot E_f \cdot A_f + N_{sd}}{0.8 \cdot 0.85 \cdot t \cdot f_{md}} = \frac{0.402 \cdot 10^{-3} \cdot 240 \cdot 10^3 \frac{N}{mm^2} \cdot 170\,mm^2 + 200 \cdot 10^3 N}{0.8 \cdot 0.85 \cdot 300\,mm \cdot 2\,N/mm^2}$$

$$\doteq 530.4\,mm$$

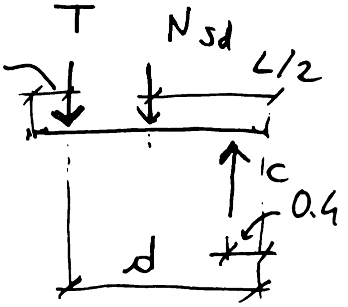
Resisting moment M_{Rd}

Considering the section's centre as pole

$$M_{Rd} = C\cdot\left(\frac{L}{2} - 0.4x\right) + T\cdot\left(\frac{L}{2} - (L-d)\right)$$

$$\overset{!}{=} \frac{1}{2} C\,(L - 0.8x) + \frac{1}{2} T\,(2d - L)$$

$$\overset{!}{=} \frac{1}{2}\left[C(L-0.8x) + T(2d-L)\right]$$



$$C = 0.85 \cdot 2\,\frac{N}{mm^2} \cdot 0.8 \cdot 530.4\,mm \cdot 300\,mm = 216.4\ kN$$

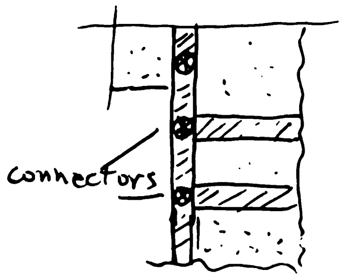
$$T = 0.402 \cdot \cancel{10^{-3}} \cdot 240 \cdot \cancel{10^{3}}\,\frac{N}{mm^2} \cdot 170\,mm^2 = 16.4\ kN$$

$$M_{Rd} = 0.5 \cdot \left[216.4\ kN\,(2 - 0.8 \cdot 0.5304)\,m + 16.4\ kN \cdot (2 \cdot 1.875 - 2)\,m\right]$$

$$\overset{!}{=} 154.1\ kN\cdot m < 250\ kN\cdot m = M_{sd}$$

M_{Rd} is not sufficient.

Possible solutions:
- increase FRP area
- adopt connectors to increase max ε_f



Suppose to adopt connectors. You could assume a larger deformation for the FRP, since debonding is prevented or put back

$\varepsilon_{fd} = 3$ ‰ seems reasonable, even if you should demonstrate it (by means of experimental tests), since there aren't wide investigations or models about connectors.

$$\varepsilon_{fd} = 3‰ \rightarrow x = \frac{3\cdot 10^{-3} \cdot 240 \cdot 10^{3}\,\cancel{N/mm^2} \cdot 170\,\cancel{mm^2} + 200 \cdot 10^{3}\,\cancel{N}}{0.8 \cdot 0.85 \cdot 2\,\cancel{N/mm^2} \cdot 300\,\cancel{mm}}$$

$$\overset{!}{=} 790.2\ mm$$

$M_{Rd} = 327.6\ kN$ $\quad (C = 322.4\ kN,\ T = 122.4\ kN)$

$M_{Rd} > M_{sd}$

Shear verification

$V_{sd} = 100\ kN$ (constant along the height)

$$V_{Rd} = \min\{\underbrace{V_{Rd,m}}_{\text{shear resistance of masonry}} + \underbrace{V_{Rd,f}}_{\text{shear resistance of FRP}} ; \underbrace{V_{Rd,max}}_{\text{strength of the compressed masonry strut}}\} \quad (\text{Eq } 5.16)$$

$$V_{Rd,m} = \frac{1}{\gamma_{Rd}} \cdot d \cdot t \cdot f_{vd} \longrightarrow \frac{f_{vk}}{\gamma_M} = \frac{f_{vk0} + \sigma \cdot \mu}{\gamma_M} \quad (5.17)$$

$\sigma = \frac{N_{sd}}{d \cdot t}$

γ_{Rd}: partial factor for this mechanism, $\gamma_{Rd} = 1.20$

d: distance between the compressed edge and the FRP axis

t: wall thickness

$$V_{Rd,f} = \frac{1}{\gamma_{Rd}} \frac{0.6\, d\, A_{fw} \cdot f_{fd}}{p_f} \quad (5.18)$$

A_{fw} = FRP area parallel to the shear force

f_{fd} = design strength of FRP : $f_{fd} = \varepsilon_{fd} \cdot E_f$

p_f = spacing among shear reinforcement

$$V_{Rd,max} = 0.3 \cdot f^h_{md} \cdot t \cdot d \quad (5.19)$$

f^h_{md}: design masonry compressive strength parallel to mortar joints

CNR DT 200/2004 § 5.4.1.2.2 deals with vertical and horizontal FRP configurations (both flexural and shear reinforcements), symmetrically applied.

$V_{Rd,m} \rightarrow$ the same for steel reinforced masonry (Italian Building Code, § 7.8.3.2.2, Eq 7.8.7)
Eurocode 6 considers L (total length of the wall) instead of d (§ 6.7.2, Eq 6.35-35)

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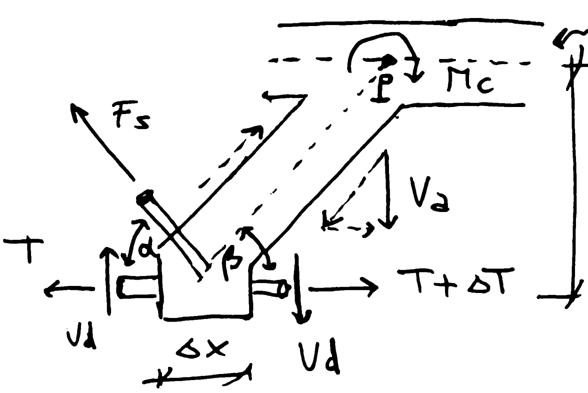
$V_{Rd,f}$ → again, the same for steel reinforced masonry (NTC §7.8.3.2.2 Eq 7.8.9)
EC6 adopt 0.9 instead of 0.6

$V_{Rd,max}$ → the Italian Building Code (Eq 7.8.10) adopts f_d (vertical strength) instead of f_d^h (horizontal strength)

Summing up: FRP shear model proposed by CNR DT200 (2004) is based on steel reinforced masonry models that are based on reinforced concrete theory (Moersch truss)

before Italian Building Code 2008

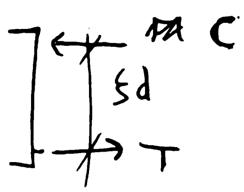
Moersch truss for a R.C. beam (taken from Prof. Giannini's notes) [only bending, no compression]



Rotational equilibrium of the concrete strut around P

$$\Delta T \cdot \xi d - V_a \cdot \Delta x - V_d \cdot \Delta x - M_c - F_s \cos\alpha \, \xi d +$$
$$- F_s \operatorname{sen}\alpha \cdot \left(\frac{\xi d}{\operatorname{sen}\beta} \cdot \cos\beta \right) = 0$$

$$\Delta T \, \xi d - (V_a + V_d) \Delta x - M_c - F_s (\cos\alpha + \sin\alpha \cot\beta) = 0$$



In the case of only bending, the moment doesn't depend on a pole → $M = T \cdot \xi d$

$$\Delta T = \frac{\Delta M}{\xi d}$$

ΔT = variation of tensile force on the flexural steel reinforcement between two sections spaced by Δx

Shear force $V = \lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x}$

ΔM = variation of bending moment dividing by Δx:

$$\frac{\Delta M}{\Delta x} - (V_a + V_d) - \frac{M_c}{\Delta x} - \frac{F_s}{\Delta x}(\cos\alpha + \sin\alpha\cot\beta)\,\xi d = 0$$

$$\underbrace{V}_{} = \underbrace{V_a + V_d + \frac{M_c}{\Delta x}}_{\text{shear carried by concrete in absence of specific rebar } V_{Rd1}} + \underbrace{\frac{F_s}{\Delta x}(\cos\alpha + \sin\alpha\cot\beta)\,\xi d}_{\text{steel contribution}}$$

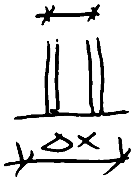
→ Considering stirrups: $\alpha = 90° \rightarrow \cos\alpha = 0,\ \sin\alpha = 1$

adopting $\beta = 45° \longrightarrow \cot\beta = \frac{\cos\beta}{\sin\beta} = 1$

approximating $\xi d \cong 0.9\,d$

considering $F_{s,max} = f_y \cdot A_{steel}$

s = stirrups spacing



$$V_{Rd} = V_{Rd1} + \frac{f_y \cdot A_{steel}}{\Delta x} \cdot 0.9\,d$$

A_{steel} = total area of steel = number of stirrups · $\underbrace{\text{area of one stirrup}}_{\text{both branches}}$

$$n = \frac{\Delta x}{s} \longrightarrow A_{steel} = \frac{\Delta x}{s} \cdot A_{sw}$$

$$V_{Rd} = V_{Rd1} + \frac{0.9\,d \cdot f_y \cdot A_{sw}}{s}$$

Same concept for steel reinforced masonry and FRP reinforced masonry: consider the contribution of one material and add the reinforcement contribution.

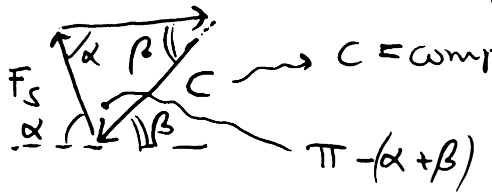
Models for shear are a difficult issue and there is still a low grade of agreement also for RC.

Note for steel reinforced masonry the Italian code proposes 0.6 instead of 0.9
This reduction could be justified that it has been experimentally observed that the horizontal reinforcement (→ "stirrups") does not reach yielding (so f_y is reduced to something like $c \cdot f_y$)

Limit: failure of the compressed strut

Recalling the Moersch truss for a reinforced concrete beam:

ΔT ⇝ tensile force increment on the flexural reinforcement

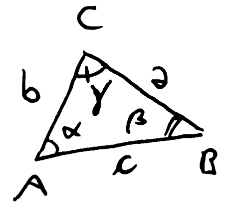


⇝ C = compression on the concrete strut

F_s = force on the steel shear reinforcem.

Sine theorem

(law of sines)

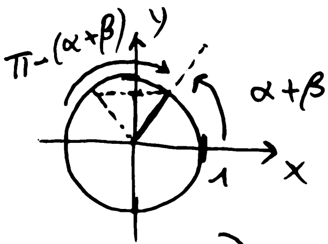


$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = \frac{abc}{2S} = 2R$$

S = triangle's area

R = circumcircle's radius

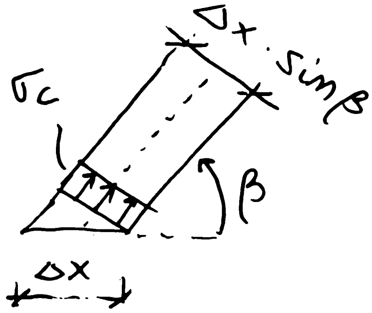
$$\frac{C}{\sin\alpha} = \frac{\Delta T}{\underbrace{\sin(\pi-\alpha-\beta)}_{\text{equal to } \sin(\alpha+\beta)}}$$



$$\frac{C}{\Delta T} = \frac{\sin\alpha}{\sin(\alpha+\beta)}$$

but $\Delta T = \dfrac{\Delta M}{\xi d}$ ⇝ bending moment variation; ξd: lever arm of internal forces

$$C = \frac{\Delta M}{\xi d} \cdot \frac{\sin\alpha}{\sin(\alpha+\beta)}$$



$$C = \sigma_c \cdot \underbrace{b}_{\text{width of the beam}} \cdot \Delta x \sin\beta$$

$$\sigma_c \, b \, \Delta x \sin\beta = \frac{\Delta M}{\xi d} \, \frac{\sin\alpha}{\sin(\alpha+\beta)}$$ ← divide it by Δx

$$\sigma_c \, b \sin\beta = \frac{\Delta M}{\Delta x} \, \frac{1}{\xi d} \cdot \frac{\sin\alpha}{\sin(\alpha+\beta)}$$

$$\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = V \longrightarrow V = \xi d \cdot \sigma_c \, b \sin\beta \cdot \frac{\sin(\alpha+\beta)}{\sin\alpha}$$

sine addition $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$V = \sigma_c \, b \cdot \xi d \cdot \sin\beta \, (\cos\beta + \cot\alpha \cdot \sin\beta)$$

$\underbrace{\max \sigma_c} \longrightarrow \max V$

it could be f_c (compressive strength of concrete) but, in practice, the adopted scheme is too simplified and compressive stresses are not equally distributed on the concrete strut section $\longrightarrow$ strength reduction $\sigma_{max} = \nu f_{cd}$

Old Eurocod 2 proposed $\nu = \max\left\{0.7 - \frac{f_{cd}}{200} ; 0.5\right\}$

$$V_{Rd,max} = \nu f_{cd} \cdot b \cdot \xi \cdot d \cdot \sin\beta \left(\cos\beta + \cot\alpha \sin\beta \right)$$

stirrups $\longrightarrow \alpha = 90°$
$\beta = 45°$ ~~always~~

$$V_{Rd,max} = \nu f_{cd} \cdot b \cdot \xi \cdot d \cdot \frac{1}{2} \overset{!}{=} \frac{1}{2} \nu f_{cd} \cdot b \cdot \xi d$$

Eq (5.19) § 5.4.1.2.2 CNR DT 200/2004: $V_{Rd,max} = 0.3 f^{h}_{md} \cdot d \cdot t$

If you consider $\nu = 1$ and $\xi = 0.6$ you obtain 0.3 that is, the same formula to calculate the masonry compressed strut strength [$b \equiv t$!]

Moreover: if you adopt a lever arm $\xi d = 0.6\,d$ instead of $0.9\,d$ adopted for concrete and steel reinforced masonry, you will obtain the eq. (5.18) without introducing any reduction coefficient.

My opinion: contribution of steel reinforcement, for masonry, is based on RC theory (approximate lever arm of internal forces is about $0.9\,d$), but it's reduced because steel doesn't reach yielding. On the other hand, for FRP reinforced masonry you consider the debonding strength of the reinforcement, which it can probably reach, ~~but~~ without reduction, but it was (maybe) observed that compressed area of masonry ~~and~~ is probably larger than in the case of steel reinforced masonry

So, it was probably adopted an approximate value of $0.6 \cdot d$ for the lever arm of internal forces, in the case of FRP reinforced masonry

Note that EC2 and NTC 2008 provide different models for shear of concrete

———— $V_{sd} = 100\ kN$

$$V_{Rd,m} = \frac{1}{\gamma_{Rd}} \left(d \cdot t \cdot f_{vd,0} + \frac{\mu}{\gamma_n} \cdot \frac{N_{sd}}{d \cdot t} \cdot d \cdot t \right)$$

$$= \frac{1}{1.2} \left(1875 \cdot 300 \cdot 0.075\ N + \frac{0.4}{2} \cdot 200 \cdot 10^3 N \right)$$

$$= 68.49\ kN$$

horizontal reinforcement!

$$V_{Rd,f} = \frac{1}{\gamma_{Rd}}\ 0.6 \cdot \frac{A_f \cdot E_f \cdot \varepsilon_{fd}}{p_f} \cdot d \qquad \varepsilon_{fd} = 0.569\ ‰$$

$$= \frac{1}{1.2} \cdot \frac{0.6 \cdot 185\ mm^2 \cdot 240 \cdot 10^3\ N/mm^2 \cdot 0.569 \cdot 10^{-3} \cdot 1875}{10^3}$$

$$= 10.88\ kN$$

$$V_{Rd} = V_{Rd,m} + V_{Rd,f} = 68.49 + 10.88\ kN = 79.37 < V_{Sd}$$

Solutions → ~~[illegible]~~ increasing FRP area
→ adopting connectors

connectors → try with $\varepsilon_{fd} = 3‰$ (you should make some tests!)

$$V_{Rd,f2} = V_{Rd,f1} \cdot \frac{3}{0.569} = 57.36\ kN$$

$$V_{Rd,2} = 68.49 + 57.36 = 125.85\ kN > V_{sd} = 100\ kN$$

CNR DT 200/Rev1 2012: the formulation for shear strength evaluation is slightly changed

Eq 5.21 (2012) $\quad V_{Rd,m} = x \cdot t \cdot f_{vd}$

change: x (distance of neutral axis from the compressed edge) instead of d (distance of the flexural reinforcement from the compressed edge)

Note: Italian Building code (2008), for steel reinforced masonry, still considers d

In practice, CNR Rev 2012 ~~adopts~~ considers the actual compressed masonry area $(x \cdot t)$ as resisting against shear (f_{vd} is evaluated using $\sigma = \frac{N_{sd}}{x \cdot t}$)

Eq. 5.22 (2012) $\quad V_{Rd,f} = \frac{1}{\gamma_{Rd}} \, 0.6 d \left(E_f \cdot \varepsilon_{fd} \right) \cdot 2 t_f \cdot \frac{b_f}{p_f}$

No significant change: simply, $E_f \cdot \varepsilon_{fd}$ replaced f_{fd} (that is the same thing, cause FRP are treated as linearly elastic) and $2 \cdot t_f \cdot b_f$ replaced A_{fw} (so it's more explicit that this model refers to reinforcement simmetrically applied, since $t_f \, b_f$ is the area of a single shear reinforcement)

$V_{Rd,max} = 0.3 \, f^h_{md} \cdot t \cdot d \longrightarrow$ no change

(anyway, the Italian Building code, for steel reinforced masonry, uses f_{md} (design masonry strengt used for vertical loads) instead of f^h_{md} (strength parallel to mortar joints)

Bottom section

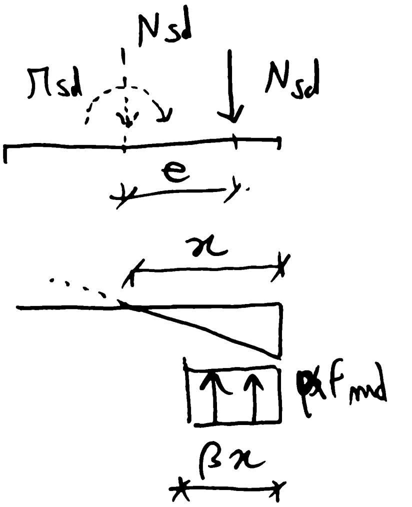
$N_{sd,2} = 243.2$ kN

$M_{sd,2} = 150$ kN·m

$V_{sd,2} = 100$ kN

$$e = \frac{M_{sd,2}}{N_{sd,2}} = \frac{150 \text{ kNm}}{243.2 \text{ kN}} = 0.6168 \text{ m} < \frac{L}{2} = 1 \text{ m}$$

The bottom section could be not reinforced against combined compression and bending



Considering only masonry (possible FRP not active)

$$C = N_{sd} = \underbrace{\alpha f_{md}}_{\text{stress}} \cdot \underbrace{\beta x \cdot t}_{\text{area}}$$

Stress block with $\alpha = 0.85$, $\beta = 0.8$

$$C = 243.2 \cdot 10^3 = 0.85 \cdot 2 \frac{N}{mm^2} \cdot 0.8 x \cdot 300\,\cancel{mm}$$

$$x = \frac{243.2 \cdot 10^3\,\cancel{N}}{0.8 \cdot 0.85 \cdot 2 \cdot 300\,\cancel{\frac{N}{mm}}} = 596.1\ mm$$

$$M_{Rd} = C \cdot \left(\frac{L}{2} - \underbrace{0.4}_{\beta/2} x\right) = 243.2\ kN \left(1m - 0.4 \cdot 596.1 \cdot 10^{-3} m\right)$$

$$= 185.2\ kN \cdot m > M_{sd,2} = 150\ kN \cdot m$$

<u>shear</u> at bottom section

$$V_{Rd,m} = f_{vd} \cdot x \cdot t$$

Now you consider the compressed area, because no reinforcement is supposed to be applied

$$= \left(\frac{f_{vk,0}}{\gamma_M} + \frac{1}{\gamma_M}\left(\mu \cdot \frac{N_{sd}}{x \cdot t}\right)\right) \cdot x \cdot t$$

again, only the compressed section

$$= \frac{0.15}{2} \frac{N}{\cancel{mm^2}} \cdot x \cdot 300\,\cancel{mm} + \frac{0.4}{2} \cdot 243.2 \cdot 10^3\,N$$

$= \rightarrow$ find the actual x, not the previous one evaluated at ultimate conditions

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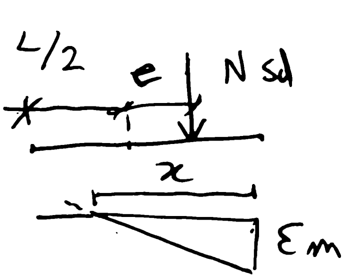

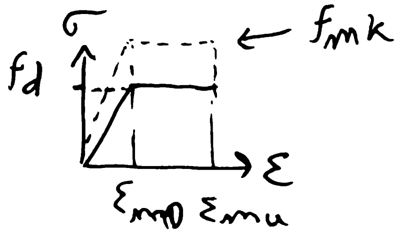

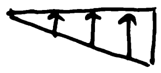

Hp: $\varepsilon_m \leq \varepsilon_{m0} = \dfrac{f_{mk}}{E_m} = \dfrac{4\ N/mm^2}{4 \cdot 10^3\ N/mm^2} = 1‰$

$$E_m' = \frac{f_{md}}{\varepsilon_{m0}} = \frac{2\ N/mm^2}{10^{-3}} = 2000\ MPa$$

If $\varepsilon_m \leq \varepsilon_{m0}$, stresses are linear $\rightarrow \left(\frac{L}{2} - e\right) = \frac{1}{3} x$

$$x = 3\left(\frac{L}{2} - e\right) = 3\left(10^3 - 616.8\right)\ mm = 1149.6\ mm$$

$$N = \frac{1}{2} \sigma_m (x \cdot t) \rightarrow \sigma_m = \frac{2N}{x \cdot t} = \frac{2 \cdot 243.2 \cdot 10^3\ N}{1149.6\ mm \cdot 300\ mm}$$

$$\stackrel{!}{=} 1.41\ N/mm^2 \leq f_{md} \longrightarrow \sigma_m / E_m' = \frac{1.41}{2 \cdot 10^3} = 0.7‰ \leq \varepsilon_{m0}$$

The assumption is confirmed $[\varepsilon_m \leq \varepsilon_{m0}] \rightarrow x = 1149.6\ mm$

$$V_{Rd,m} = 0.075 \cdot 300 \cdot 1149.6 + 0.2 \cdot 243.2 \cdot 10^3\ N$$
$$\stackrel{!}{=} 74.5\ kN < V_{Sd,2} = 100\ kN$$

Therefore shear reinforcement is necessary
If you suppose to extend the vertical reinforcement beyond the base section and to apply horizontal reinforcement, you obtain an overdesigned flexural strength but you can apply the FRP resisting model for shear

$$V_{Rdm} = \frac{1}{\gamma_{Rd}} \cdot \left(d \cdot t \cdot f_{vd0} + \frac{0.4}{\gamma_M} \cdot N_{sd,2}\right) = \frac{1}{1.2}\left(1875 \cdot 300 \cdot 0.075 + 0.4/2 \cdot 243.2 \cdot 10^3\right) N$$
$$\stackrel{!}{=} 75.7\ kN$$

$$V_{Rd,f} = \frac{1}{\gamma_{Rd}} \cdot 0.6 \cdot \frac{A_{fw} \cdot E_f \cdot \varepsilon_{fd}}{p_f} \cdot d = 57.36\ kN$$

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$V_{Rd} = V_{Rdm} + V_{Rdf}$

$\overset{!}{=} 75.7\ kN + 57.36\ kN = 133.06\ kN > V_{Sd_2} = 100\ kN$