FRP REINFORCEMENT OF MASONRY

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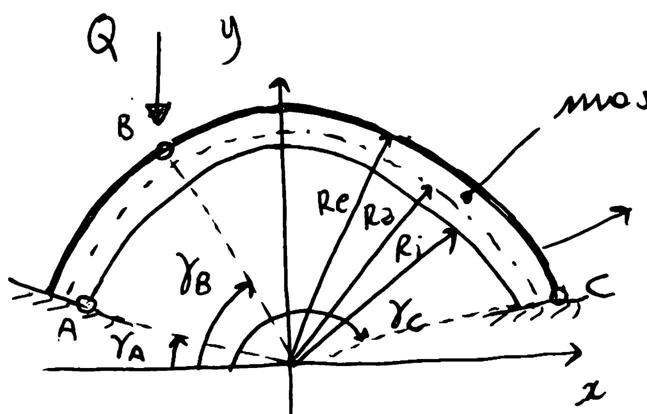
Padova, 5/7/2012

NOTES ABOUT THE ASSIGNMENT by MATTEO PANIZZA



Radii

$R_i \rightarrow$ intrados

$R_a \rightarrow$ axis

$R_e \rightarrow$ extrados

Angles

$\gamma_A = 180° - \gamma_C = 34° \rightarrow \gamma_C = 146°$

$\gamma_B = 65.5°$

$R_i = 903$ mm

$s = 70$ mm (thickness)

$R_a = R_i + s/2$

$R_e = R_i + s$

width $b = 1000$ mm

Specific weight $\rho = 18\ kN/m^3$

Masonry properties

$E_m = 3 \cdot 10^3\ N/mm^2$ (Young's Modulus)

$f_{mk} = 4.3\ N/mm^2$ (characteristic compressive strength)

$\varepsilon_{mu} = 3.5‰$ (ultimate strain)

Suppose to apply Carbon fabrics with epoxy (CFRP)

$E_f = 230$ GPa (tensile elastic modulus)

$\varepsilon_{fk} = 2.1\%$ (characteristic ultimate strain)

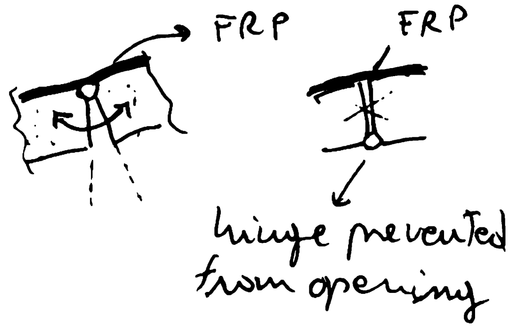
$t_f = 0.17$ mm (equivalent thickness)

$\rightarrow f_{fk} = E_f \cdot \varepsilon_{fk} = 4830\ N/mm^2$

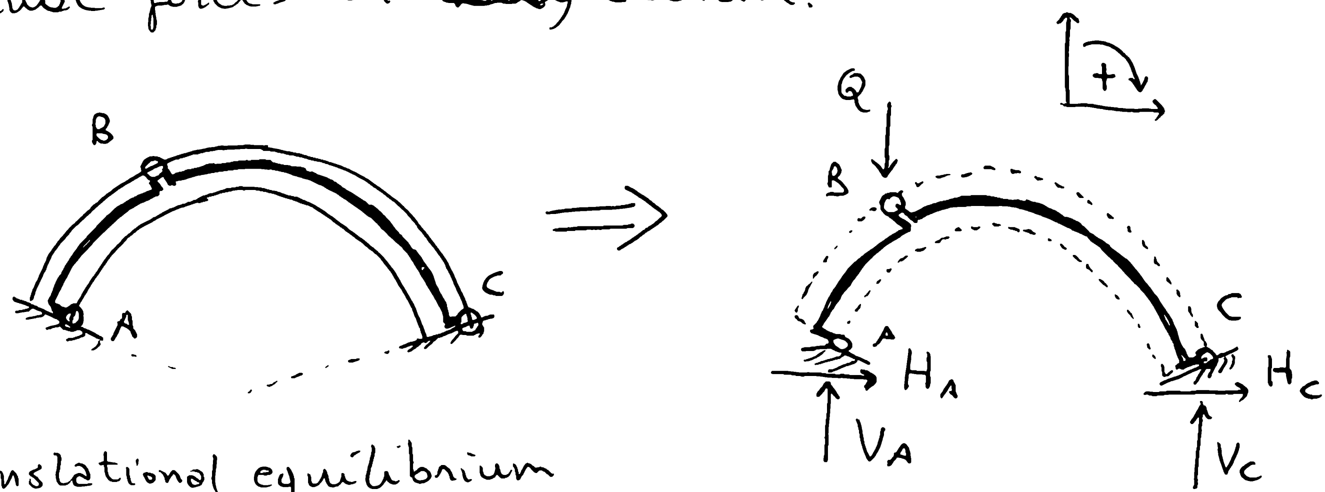
External load $Q = 6.5$ kN

Suppose that the arch needs to be strengthened to withstand the load Q (don't evaluate the UR arch strength) and that the arch will crack (that is, hinges open) under Q.

Along the arch, only hinges on the same side where FRP is applied (except at the springers, where FRP ends!)



Following the simplified approach proposed by Valluzzi et al. (2001), adopt the same hinges' positions related to the unreinforced arch, excluding the hinge comprised between B and C, which would be the fourth one that renders the arch labile. Having just three possible hinges that you suppose open after a certain load level lower than Q (simplification), the arch becomes statically determined, so you are able to evaluate reactions at the springers and internal forces at ~~every~~ each section.



Translational equilibrium

$$V_A + V_C = Q + W \longrightarrow \text{arch self weight}$$

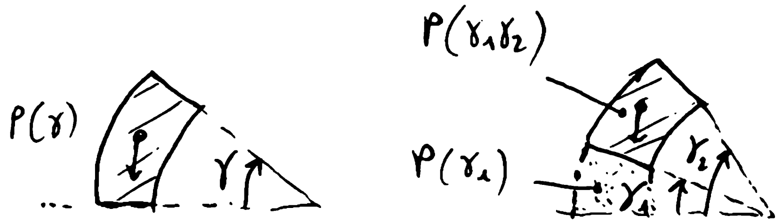
$$H_A + H_C = 0$$

Rotational equilibrium $\longrightarrow$ $(H_A; V_A)$ and $(H_C; V_C)$

Self weight per unit width : $W(\gamma) = \rho \cdot \frac{\gamma}{2}\left(R_e^2 - R_i^2\right)$

↳ pay attention on how you calculate γ!

γ = angle in radians!



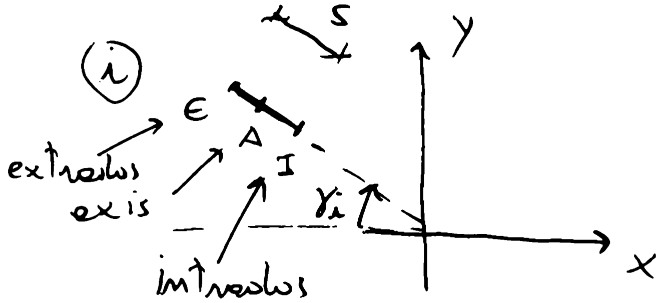
$$P(\gamma_1\gamma_2) = P(\gamma_2) - P(\gamma_1)$$

$$W = b\cdot\left(P(\gamma_c) - P(\gamma_A)\right) = b\cdot \rho \cdot \frac{\gamma_c - \gamma_A}{2}\,(R_e^2 - R_i^2)$$

(γ in radians)

Once you find reactions at the springer, you can evaluate the internal forces. Discretize the structure, for istance choosing cross sections every 0.5° or 1°.

- write the geometrical coordinates of the three main points of a section (intrados, axis, extrados)



$$x_i = -R_i \cos\gamma_i$$
$$y_i = R_i \sin\gamma_i$$

and so on :.. (x_a, y_a); $(x_e; y_e)$

- Calculate the arch weight at γ_i $\quad W_i = W(\gamma_i) - W(\gamma_A)$ that is the weight of a portion between the springer A and a certain angle γ_i
- Calculate the barycentre coordinates of that portion of arch : $(x_{g0i}, y_{g0i}) \rightarrow$ portion from $\gamma = 0$ and γ_i

$(x_{gi}, y_{gi}) \rightarrow$ portion from γ_A and γ_i

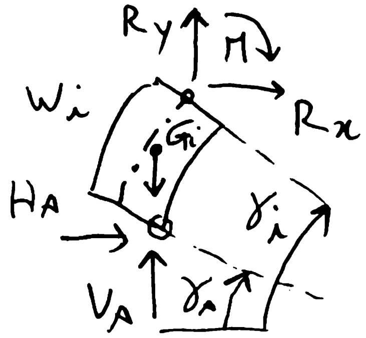
$$x_{g0i} = -\frac{4}{3}\,\frac{R_e^3 - R_i^3}{R_e^2 - R_i^2}\cdot\frac{\sin \gamma_i/2}{\gamma_i}\cos \gamma_i/2$$

$$y_{g0i} = \frac{4}{3}\,\frac{R_e^3 - R_i^3}{R_e^2 - R_i^2}\cdot\frac{\sin \gamma_i/2}{\gamma_i}\cdot\sin \gamma_i/2$$

$$x_{gi} = \frac{x_{g0i}\cdot W(\gamma_i) - x_{gA}\cdot W(\gamma_A)}{W_i}$$

$$y_{qi} = \frac{y_{g0i} \cdot W(\gamma_i) - y_{qA} \cdot W(\gamma_A)}{W_i}$$

- Now evaluate the internal forces on the generic section i with respect to the central axis R_{xi}, R_{yi} and M_i



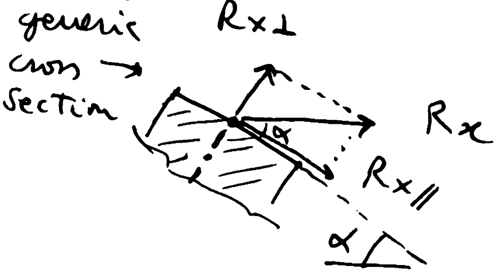
To consider the equilibrium, take into account:
- springer reactions H and V
- self weight of the arch portion W_i
- external load Q if $\gamma_i \geq \gamma_B$

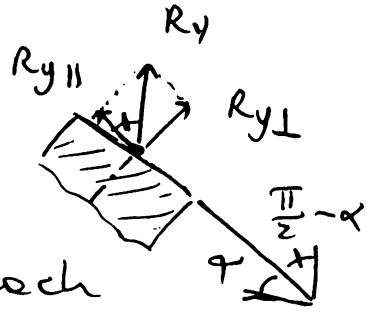
R_{xi}, R_{yi} and M_i → evaluated on the basis of a fixed reference system

Then, you should calculate N_i, V_i and M_i with respect to a standard reference system based on that section

It's simply a geometrical operation

Example

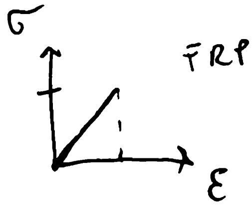


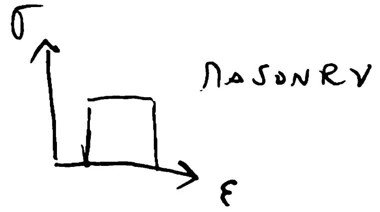


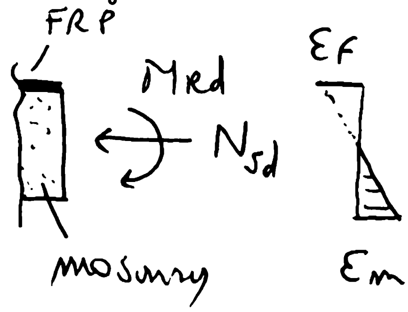
Just pay attention to the sign of each contribution, considering that N is negative in the case of compression force, V is positive if related to an ideal clockwise rotation of a small element of structure, M is positive when "inferior fibres" are under tension → for the sake of simplicity, when ~~FRP~~ ~~is~~ M tends to put in tension the side where FRP is applied

verification against masonry crushing

Adopt Triantafillou (1998) hypotheses (see also Valluzzi et al. 2001, De Lorenzis and La Tegola 2007)







FRP → linear elastic till failure

masonry → stress block

$$\omega = \frac{\varepsilon_{mu} \cdot E_f}{f_{mk}} \cdot \frac{A_f}{b \cdot s}$$

A_f → FRP area

$b \cdot s$ → area of the masonry cross section (width x thickness)

ω → normalized FRP area fraction

E_f → FRP Young's modulus $\boxed{\gamma_m = 2.5}$

ε_{mu} → ultimate masonry strain

f_{mk} → characteristic masonry compressive stress

$$\frac{M_{Rd}}{b s^2 f_{mk}} = \frac{1}{2} \omega \frac{1-\bar{x}}{\bar{x}} + \frac{0.4}{\gamma_m} \cdot \bar{x} \left(1 - 0.8 \bar{x}\right)$$

provided that $\omega \geq \omega_{lim}$

$$\omega_{lim} = \frac{\varepsilon_{mu} E_f}{f_{mk}} \cdot \rho_{lim} = \frac{\varepsilon_{mu}}{\varepsilon_{fk}} \left[\frac{0.8}{\gamma_m} \frac{1}{\left(1 + \frac{\varepsilon_{fk}}{\varepsilon_{mu}}\right)} - \frac{N_{sd}}{b s f_{mk}} \right]$$

$$\bar{x} = \frac{x}{t} = \frac{\gamma_m}{1.6} \left[\frac{N_{sd}}{b s f_{mk}} - \omega + \sqrt{\left(\omega - \frac{N_{sd}}{b s f_{mk}}\right)^2 + \frac{3.2}{\gamma_m} \omega} \right]$$

→ non-dimensional depth of the neutral axis at failure

For the sake of simplicity: consider the FRP working till failure (don't consider possible strain reduction due to debonding mechanism), anyway adopting $\varepsilon_{fd} = \eta_a \cdot \frac{\varepsilon_{fk}}{\gamma_f}$ as strain limit

Adopt a guess value for FRP width to be applied to the arch extrados

Calculate $\bar{x}$, once N_{sd} is known (you calculated it! it's the actual normal force on the cross section)

after you checked that $\omega \geq \omega_{lim}$ for each section

Evaluate M_{Rd} for each section and compare it to the M_{sd} that you calculated before.

Verify that $M_{Rd} \geq M_{sd}$ for each section

- Verification against intermediate debonding

Plate-end debonding is not likely to occur (never experimentally observed), partly because near the FRP ends the thrust line enter the cross section and the reinforcement is not working ⇒

Intermediate debonding is non considered for masonry by CNR DT 200 provisions, so adopt the formula tion provided in the case of concrete, making use of the simplified approach with $K_{cr} = 3.0$

Find the actual tensile force on the FRP (T) using Foraboschi hypothesis → fixed compressed masonry area equal to one third of the thickness also if not in ultimate conditions.

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- <u>Step 1</u> solve the structure using the given value of external load Q

  <u>Output</u>: Reaction at springers H and V → this is not shear!

- <u>Step 2</u> : find the internal forces N, V and M

  <u>output</u>: Diagrams of compressive force N, shear force V and bending moment M for the whole structure

- <u>Step 3</u> : find M_{Rd} for each section and possibly find a suitable amount of reinforcement to verify that $M_{Rd} \geq M_{sd}$

  <u>output</u> : diagram that shows M_{Rd} compared to M_{sd} for the whole structure

- <u>Step 4</u> : find the tensile force T on the FRP related to the load Q (the actual tension!), using the simplified assumptions of Foraboschi (2004)
  Compare T to Tmax, being the last one the max tensile force related to intermediate debonding (→ concrete formulation as it is)

  <u>Output</u> diagram showing T (actual) compared to Tmax for the whole structure

- <u>end</u>

Don't care too much about design/characteristic values, partial factors and so on...
Just follow a "consistent" path and write all the notes/explanations that you consider significant